PHYSICS 20

Module 3

Curved Motion and Gravitation

Distance Learning

Alberta
EDUCATION

Physics 20

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Curved Motion and Gravitation

Alberta
EDUCATION

Distance Learning

This document is intended for	
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Teachers (Physics 20)	✓
Administrators	
Parents	
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Other	

Physics 20
Student Module
Module 3
Curved Motion and Gravitation
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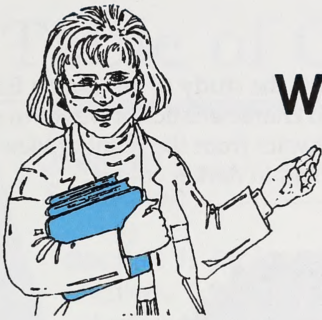
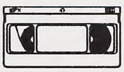


Welcome to Module 3!

We hope you'll enjoy your study of *Curved Motion and Gravitation.*

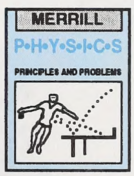
To make your learning a bit easier, watch the referenced videocassettes whenever you see this icon.



You also have the option of viewing laser videodisc clips when you see the bar codes like this one.

Frame 48350A

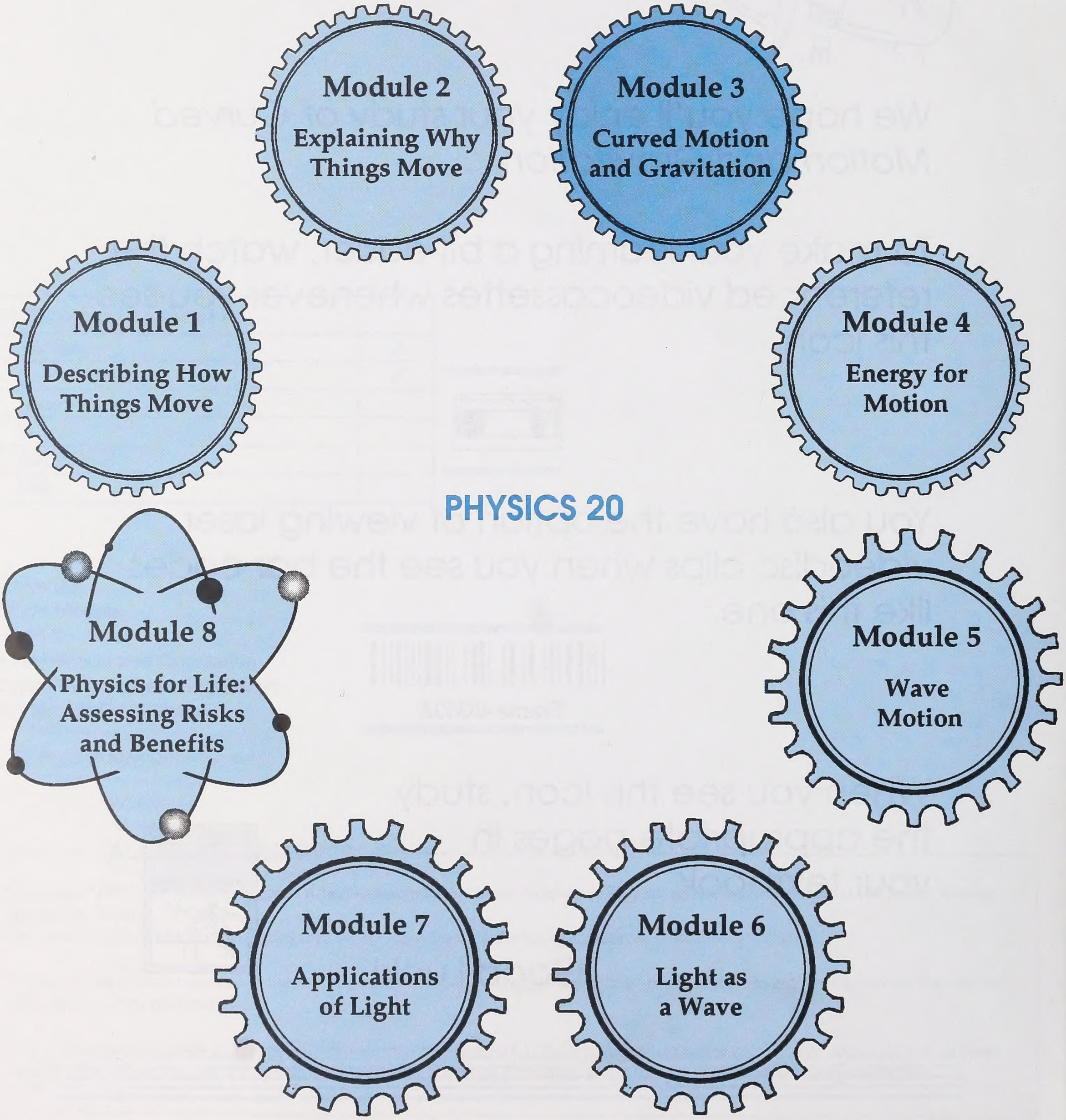
When you see this icon, study the appropriate pages in your textbook.



Good Luck!

Course Overview

This course contains eight modules. The first four modules involve the study of motion on Earth and in the heavens. Modules 5, 6, and 7 investigate the properties and characteristics of waves in general and light waves. The last module is an introduction to nuclear physics from the point of view of risk/benefit analysis. The module you are working in is highlighted in darker colour.



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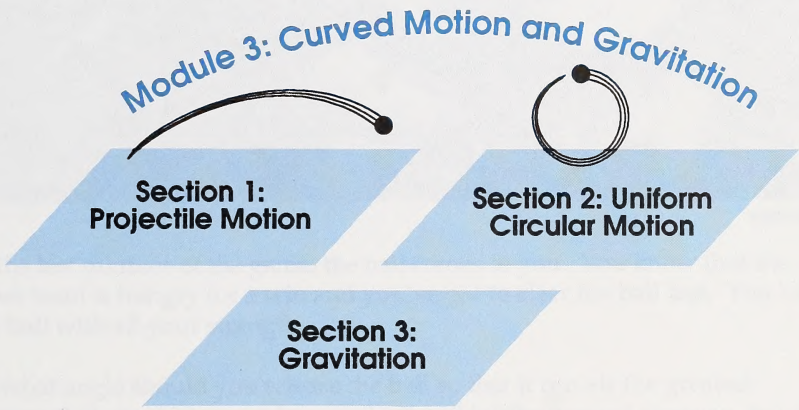
OVERVIEW

The ball is barely visible as it sails through the night sky and into your ball glove. The street lamps have just come on, so you can keep playing catch. The light from the full moon helps you see the ball and adds to the exciting atmosphere of the cool fall night.

The motion of the ball through the air and the orbit of the moon actually have a lot in common. The moon and the ball both follow curved paths that are shaped by the gravitational pull of Earth.

In this module you will use physics to develop mathematical descriptions for the path of both the ball and the moon. You will also be introduced to Newton's law of universal gravitation. This law helps explain the motion of the ball, the moon, and other objects in the heavens.



Evaluation

Your mark in this module will be determined by your work in the Assignment Booklet. You must complete all assignments. In this module you are expected to complete three section assignments. The mark distribution is as follows:

Section 1 Assignment	32 marks
Section 2 Assignment	32 marks
Section 3 Assignment	36 marks
TOTAL	**100 marks**

Section

1 Projectile Motion

WESTFILE INC.

In the last minutes of the game, the ball comes to you. You know that the other team is hungry for a win and you've got to clear the ball fast. You kick the ball with all your strength.

At what angle should you release the ball so that it travels the greatest distance? If you kick it at a low angle, it will hit the ground too soon, but if you kick it at a steep angle, it may spend too much time in the air and not travel far enough. Is there an angle that balances these two tendencies?

People who play a lot have learned the answers to these questions from experience. After hours and hours of play, they have learned what motion and what angle produces the greatest distance.

In this section you will use physics to develop answers to questions about objects travelling through the air. You will learn how the basic physics of uniform and accelerated motion applies to projectiles. You will investigate the motion of a projectile and then you will develop equations from your investigation. Finally, you will apply the equations to some everyday events.

Activity 1: Observing Projectiles

One of the most interesting ideas in Module 2 is the idea that perpendicular vectors can be treated independently of each other. When the boat in the river crossing lab travelled straight across the current, the southerly velocity of the river did not affect the easterly velocity of the boat. Although both velocities combined to give the boat a resultant southeasterly velocity when viewed from the shore, the only thing that determined how long the boat took to cross the river was the easterly velocity of the boat, not the velocity of the river.

Does this same concept hold true for a projectile? Are you able to analyse the horizontal and vertical components of the motion independently of each other? You will answer these questions by completing the following investigation.

PATHWAYS

If you have access to the laser videodisc entitled *Physics: Cinema Classics* and a laser videodisc player, do Part A. If you do not have access to the laser videodisc, do Part B.

Part A

Investigation: Projectiles in Slow Motion

Science Skills

- [ ] A. Initiating
- [x] B. Collecting
- [x] C. Organizing
- [x] D. Analysing
- [ ] E. Synthesizing
- [ ] F. Evaluating

Purpose

In this investigation you will observe the motion of two balls projected from a launcher.

Materials

You will need the following materials for this investigation:

- the laser videodisc called *Physics: Cinema Classics*
- a laser videodisc player with a bar code reader

If your laser videodisc player does not have a bar code reader, enter the frame numbers provided with the icons to search and play each sequence.

Procedure

- Load Side B of the laser videodisc into the player and press "play" to spin the disc.
- Use the bar code reader to view the background information about the apparatus.

Frames 00566 – 00732

- Use the bar code reader to watch the balls move in slow motion. Carefully observe the motion of the balls.

Frames 00750 – 01163

Observations

1. Which ball hit the floor first?

Analysis

2. What determines how long it takes for a ball to hit the floor?

Conclusions

It would seem that the horizontal speed of a ball does not affect how long the ball takes to fall to the floor. In other words, the vertical and horizontal motions are independent of each other.

End of Part A

Part B

Investigation: Projectile Pennies

Science Skills

- ☐ A. Initiating
- ☑ B. Collecting
- ☑ C. Organizing
- ☑ D. Analysing
- ☐ E. Synthesizing
- ☐ F. Evaluating

Purpose

In this investigation you will observe the motion of two pennies projected from a tabletop.

Materials

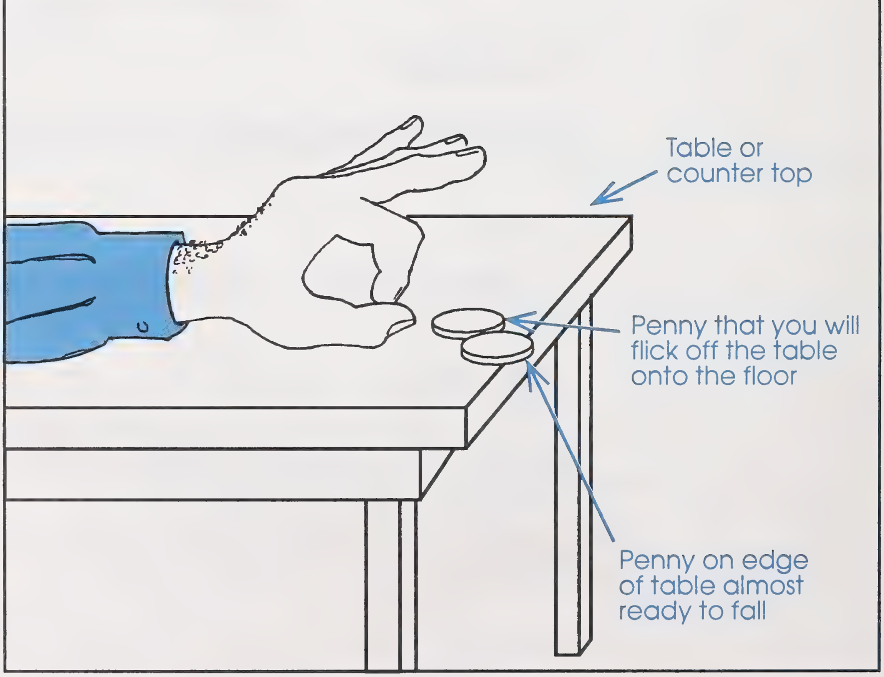
You will need the following materials for this investigation:

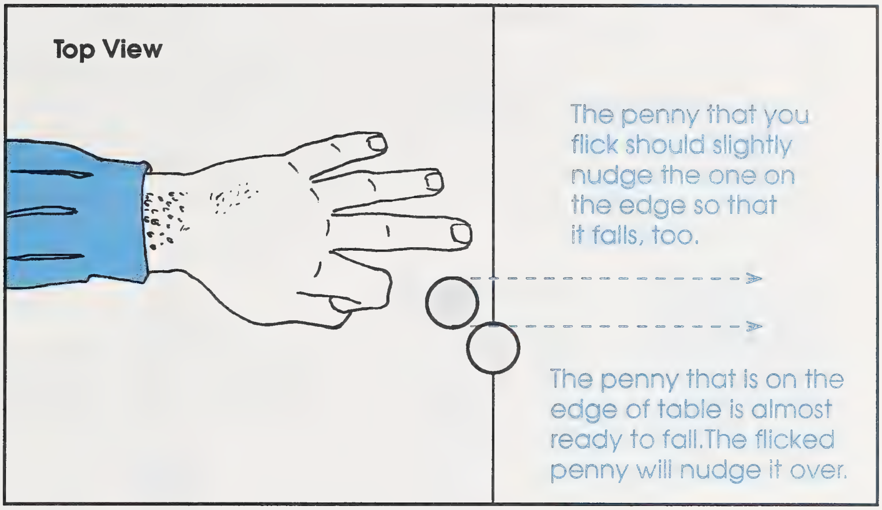
- a horizontal tabletop
- two pennies

Procedure

- Set up the apparatus as shown in the following diagrams.

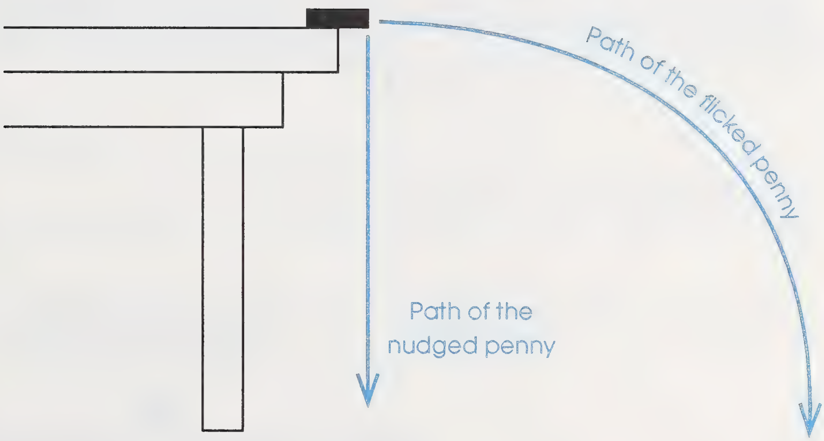
Side View





- Practise several times until you have the technique mastered.

The penny that you flick travels far from the table. The other penny is nudged over the edge and falls at the base of the table.



Observations

3. Listen to the sound of the two pennies hitting the floor. What do you notice?

4. Try flicking the penny fast and then a bit slower. Does the speed affect the time that it takes the flicked penny to hit the floor?

Analysis

5. Since the nudged penny falls straight down from the table each time, assume that it takes the same amount of time to hit the floor each time. Does the speed of the flicked penny affect how long it takes to hit the floor?

6. What determines how long it takes for either penny to reach the floor?

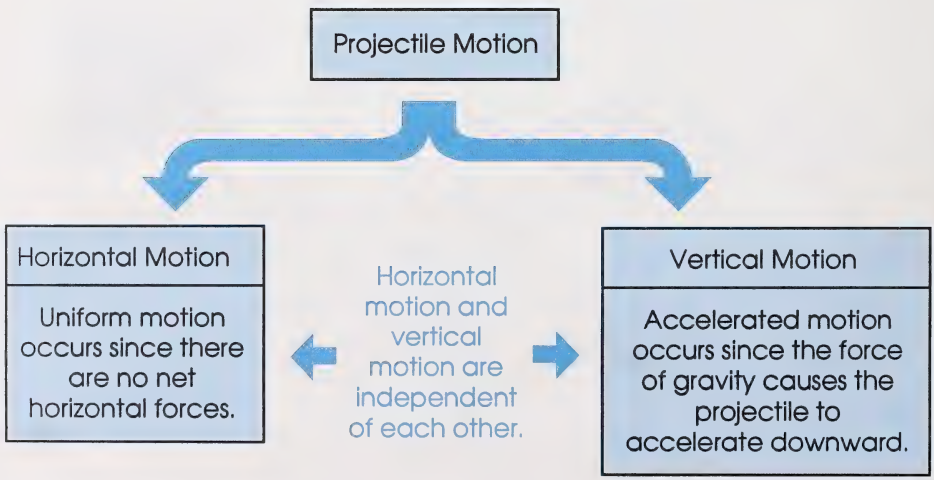
Conclusions

The horizontal speed of the pennies does not influence the time that it takes for the pennies to fall to the floor. In other words, the vertical motion and the horizontal motion are independent of each other.

End of Part B

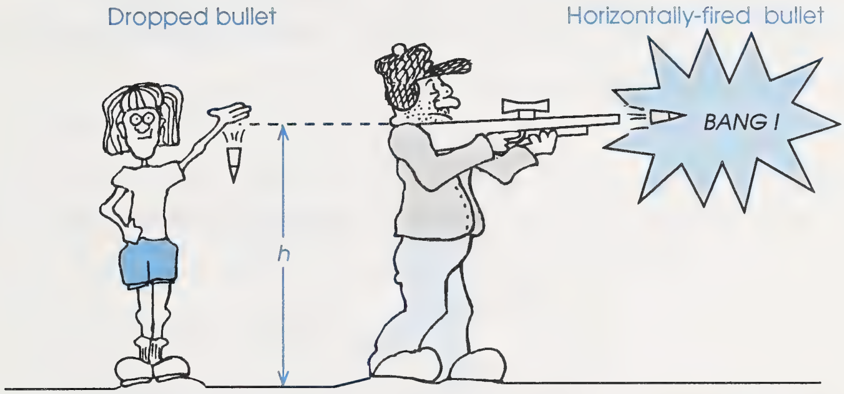
Summary

The following ideas will become the main focus of this section.



7. The preceding chart makes an assumption about air resistance. Use Newton's laws to explain that assumption.

8. Consider the following situation. Air resistance is negligible. If both bullets were released at the same time from the same height, which one would strike the ground first?



Check your answers by turning to the Appendix, Section 1: Activity 1.

Activity 2: Analysis of the Path of a Marble

Try to imagine an object moving at a constant horizontal speed while it is accelerating vertically. It is not easy to visualize, is it? The next investigation will help you understand the motion of projectiles.

Investigation: A Projectile Marble

Purpose

In this investigation you will analyse the path of a marble rolling down an inclined surface.

Science Skills

- [ ] A. Initiating
- [x] B. Collecting
- [x] C. Organizing
- [x] D. Analysing
- [x] E. Synthesizing
- [ ] F. Evaluating

Materials

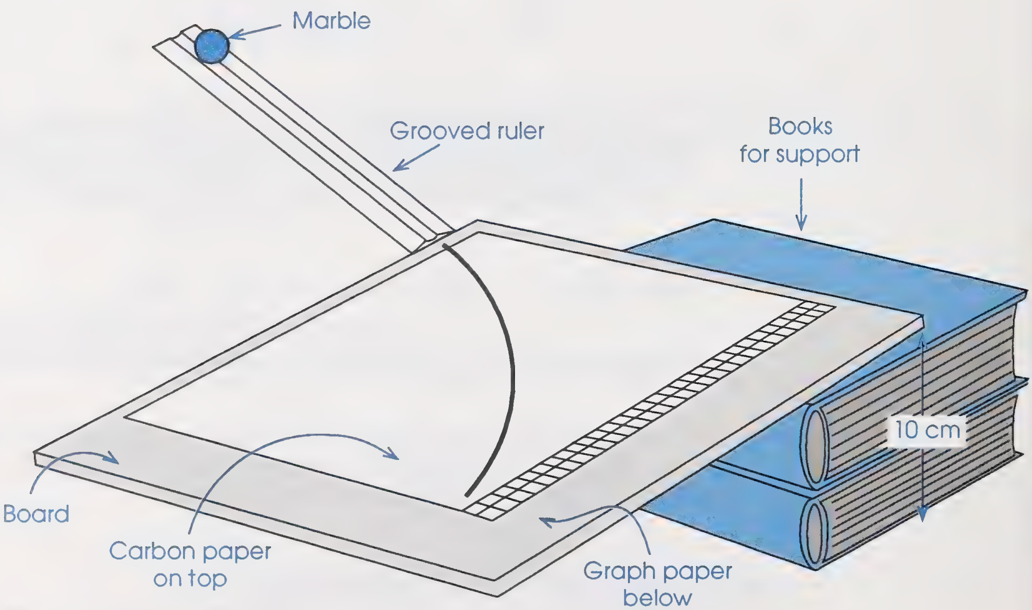
You will need the following materials for this investigation:

- large marble
- graph paper with 1-cm^2 squares (supplied in the Appendix)
- a ruler with a grooved centre
- carbon paper
- a board with minimum dimensions of 30 cm × 30 cm
- books for support

Procedure

- Set up the apparatus as illustrated in the diagram.

Side View:

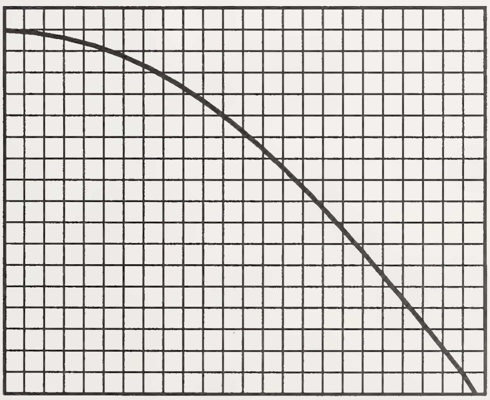


- Place one end of the board on two or three books so that the height of this end of the board is about 10 cm. Use books to support the ruler so that it is slightly higher than the high end of the board.
- Place the provided graph paper on the centre of the board. This paper is provided on a pull-out page in the Appendix.
- Position the ruler so that the groove is parallel to the top of the paper and so that the marble will roll onto the upper left corner of the paper.
- You may wish to do a trial run to make sure that the marble rolls onto the paper and makes a tracing as illustrated in the diagram.
- When you are ready to collect data, place the carbon paper on the lined paper so that the pressure of the marble on the carbon paper will trace a path onto the graph paper. Make sure that you place the carbon paper carbon-side-down on the graph paper.
- Roll the marble onto the paper.
- Retrace the path left by the marble on the graph paper with a pen or pencil. This will make the path easy to see and easy to measure.

Data

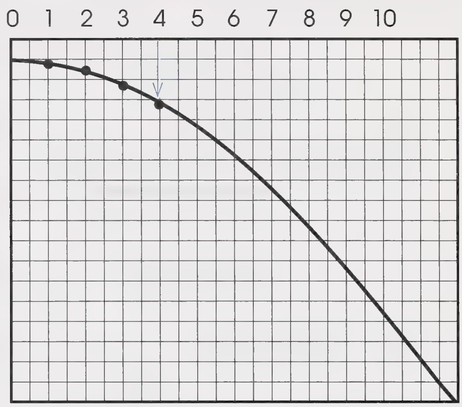
The following steps will help you to collect data from your projectile path.

- The path of the marble on your sheet of graph paper should look like this.

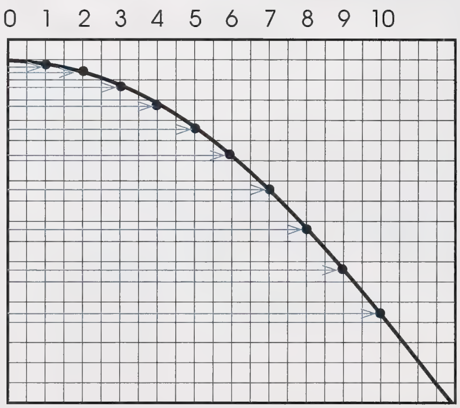


- Assume that the horizontal motion of the marble is uniform. If this is so, the horizontal spaces can be used to indicate time. Label your sheet of graph paper so that the top horizontal line becomes a time scale. Let every second line represent one unit of time (called a tock). This imaginary time unit will permit you to complete the data analysis.

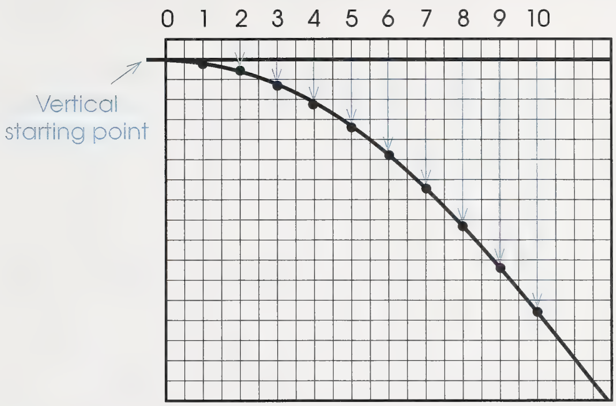
- Mark every position of the marble when it crosses a time interval line with a dot.



1. Measure the horizontal distance from the left side of the graph paper to each dot. Record your measurements in the data chart.



2. Locate the highest vertical point of the marble's path. Draw a line to indicate this starting point. Measure the vertical distance from this line to each dot. Record your measurements in the data chart.



Observations

Data for Projectile Marble				
Time Interval (tocks)	**Horizontal Distance *x* (cm)**	**Average Horizontal Speed (cm/tock)**	**Vertical Distance *y* (cm)**	**Average Vertical Speed (cm/tock)**
0				
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

Calculations

3. Use the time units and the distances on the data chart to calculate the average horizontal speed and the average vertical speed for the marble at the point represented by each dot.

   Since the horizontal motion is uniform, the following equation can be used to solve for the average horizontal speed.

   $$v_x = \frac{x}{t} \qquad \text{(horizontal distance} = x\text{)}$$

   The vertical motion is accelerated. Use the following equation to solve for the vertical speed. Note that the initial vertical speed is zero.

   $$d = \left(\frac{v_i + v_f}{2}\right)t$$

   $$y = \left(\frac{v_{yi} + v_{yf}}{2}\right)t \qquad \text{(vertical distance} = y\text{)}$$

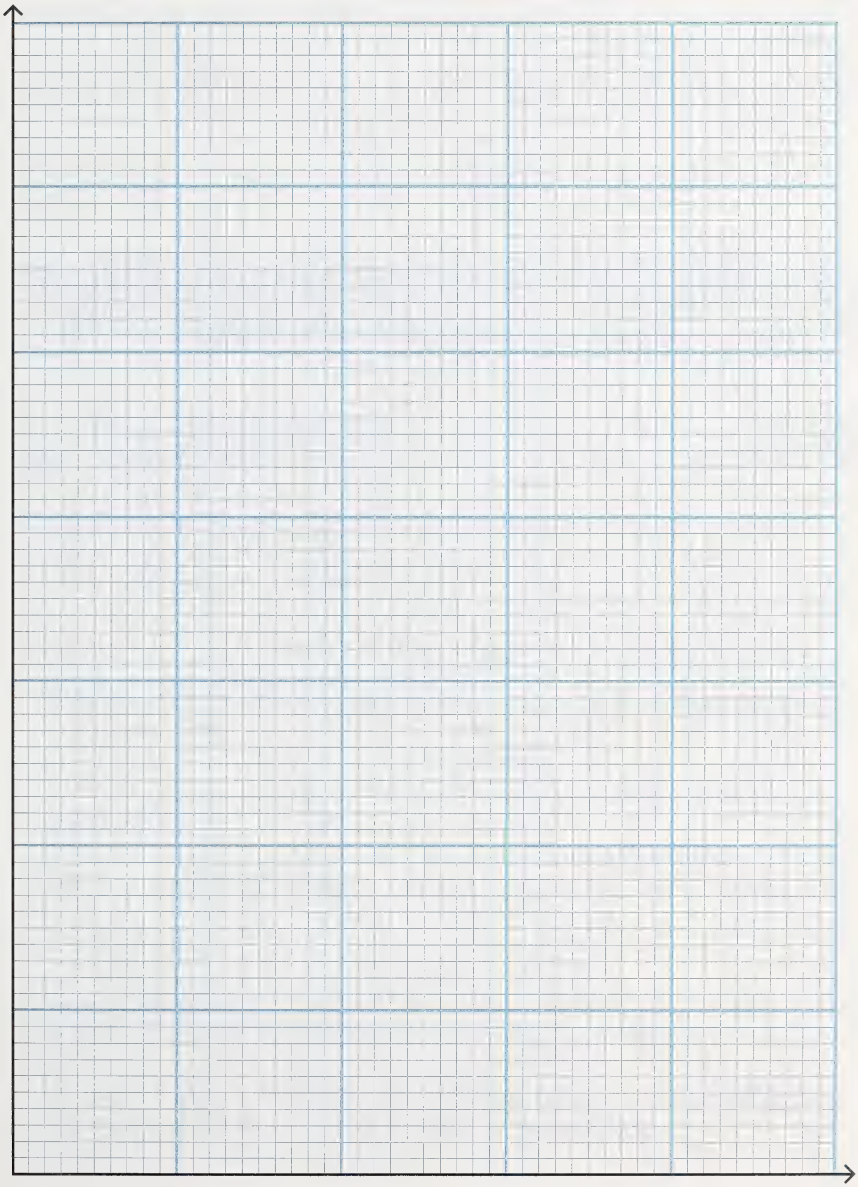
   $$y = \left(\frac{v_{yf}}{2}\right)t \qquad (v_{yi} = 0)$$

   $$v_{yf} = \frac{2y}{t}$$

   Record your calculated values for the average horizontal and vertical speeds in the data chart.

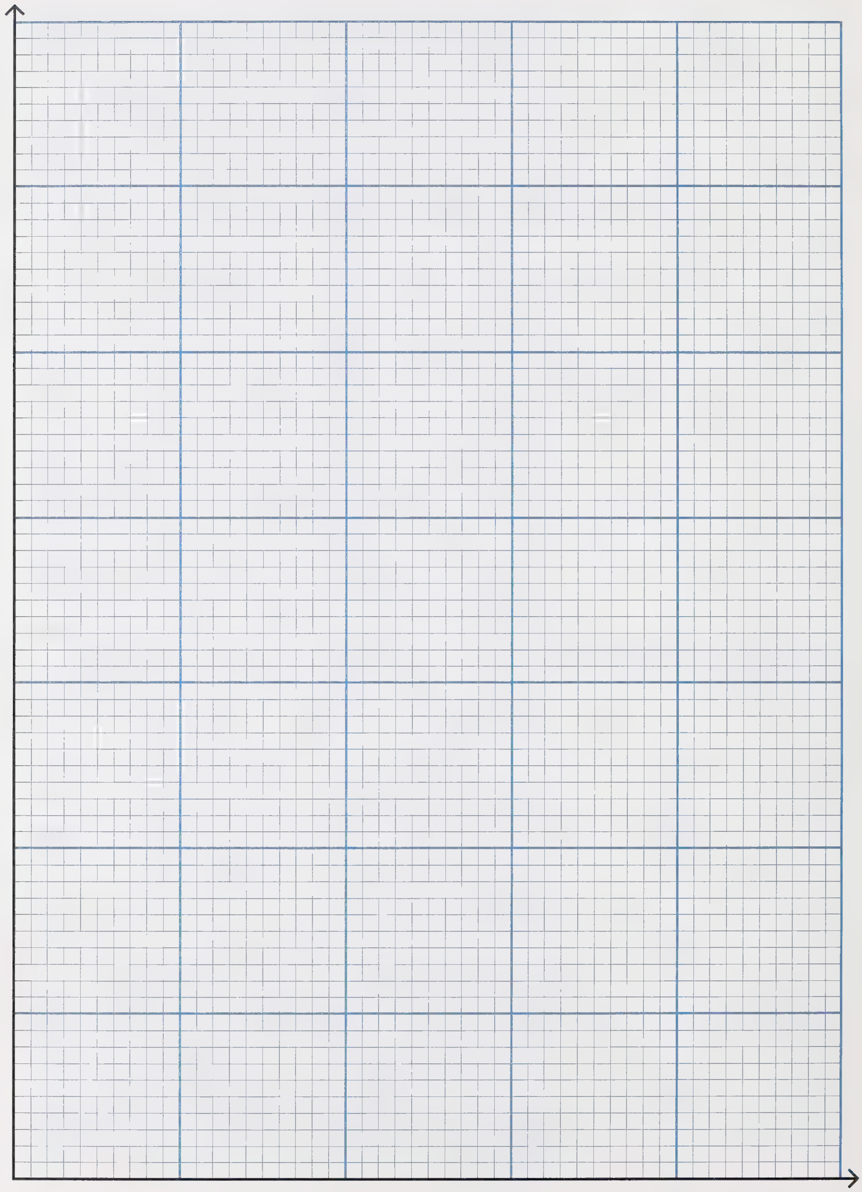
Analysis and Interpretation

4. Plot a horizontal speed versus time graph. Include a title for the graph and label the time on the x-axis and horizontal speed on the y-axis.



5. Describe the motion shown by the horizontal speed-time graph.

6. Plot a vertical speed versus time graph. Include a title for the graph and label the time on the x-axis and vertical speed on the y-axis.



7. How does the vertical motion differ from the horizontal motion of the marble?

8. Determine the slope of the vertical speed-time graph. What does this value represent?

Conclusions

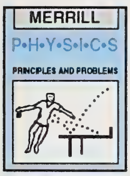
9. What conclusions can you draw about the causes of the path taken by a projectile?

In the next activity you will learn to write equations for describing the motion of projectiles.

Check your answers by turning to the Appendix, Section 1: Activity 2.

Activity 3: Equations for Projectiles

You will be glad to know that the equations needed to describe projectile motion are the same ones that you developed for uniform motion and accelerated motion in Module 1.



Read pages 132 to 135 in your textbook. Then answer questions 1 and 2.

1. Which equation is used to describe the horizontal displacement of a projectile?

2. Which equation is used to describe the vertical displacement of a projectile?

Consider the following example. Use this example to answer questions 3 to 8.

Example

A baseball is thrown horizontally at 8.0 m/s from the top of a building that is 20 m high. How far from the base of the building will the baseball land?

3. Sketch a diagram illustrating the predicted path of the baseball and the displacement of the ball from the base of the building.

4. List the two equations used to describe the horizontal and vertical motions of the baseball and then determine which variable is the same in both equations.

To solve for the horizontal distance, you must first solve for the time (t) in the equation for vertical motion.

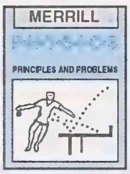
5. Manipulate the equation for vertical motion to solve for the variable t.

6. Substitute the known values into the equation.

7. Solve for the variable t.

Now that the time is known, you can substitute the time into the equation for horizontal motion and solve for the horizontal displacement.

8. Substitute the known values into the equation $\vec{x} = \vec{v}_x t$ and solve for the horizontal displacement ($\vec{x}$).

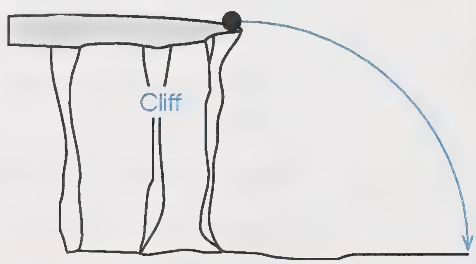


Refer to the Example Problem on page 136 in your textbook. Use the Example Problem to answer question 9.

9. a. What is the initial vertical velocity of the stone?

   b. Which equation do you use to determine the time in which the stone was in flight?

   c. On the following trajectory path of the stone, indicate the horizontal displacement of $+45\ \text{m}$.



Check your answers by turning to the Appendix, Section 1: Activity 3.

10. Now that you have examined two similar examples of projectiles launched horizontally, do Practice Problems 3 and 4 on pages 136 and 137 in your textbook.

Check your answers by turning to page 667 in your textbook.

In this activity you have described the motion of projectiles that are launched horizontally. In the next activity you will describe the motion of projectiles which are launched at some angle above the horizontal.

Activity 4: The Flight of a Soccer Ball

Not all projectiles are launched horizontally. In fact, most projectiles are launched at some angle above the horizontal. In this activity you will use the concepts of uniform motion and accelerated motion to describe the motion of projectiles launched at some angle.

1. a. Sketch the path that the soccer ball would take.

   b. What parts of the soccer ball's path are similar to the path of the marble in the previous investigation?

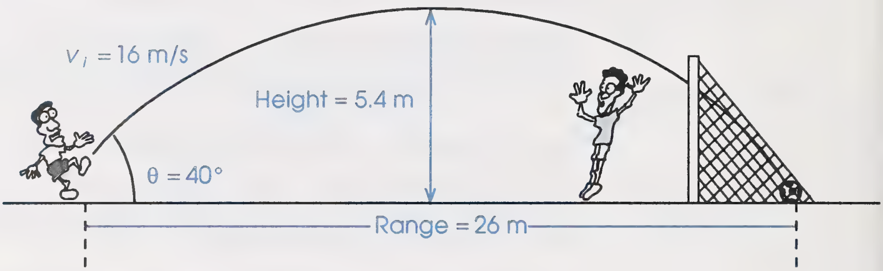
trajectory – the path followed by a projectile

range – the maximum horizontal displacement of a projectile

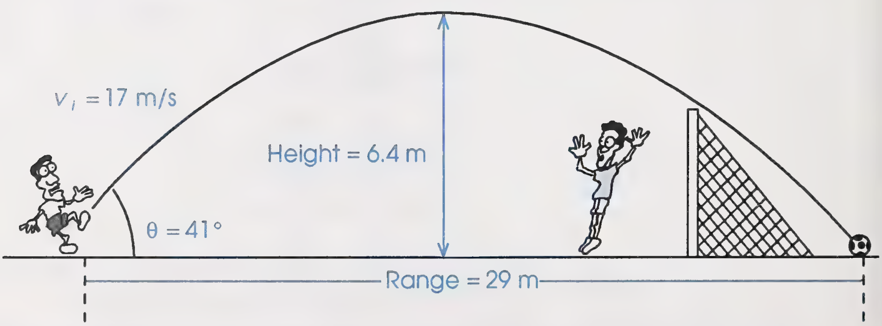
height – the maximum vertical displacement of a projectile

When soccer players kick a ball, they adjust the position of their body, especially their legs and feet, to achieve the desired **trajectory** for the ball. Sometimes a player may want to kick the ball to maximize the **range**, while at other times it might be best to focus on the **height** of the ball. Usually a player must consider both the height and the range to have the ball arrive at its target. Accurate kicking requires the player to control both the magnitude and the direction of the initial velocity that the ball has when it leaves the foot. Even the slightest variation can cause the ball to be off-target.

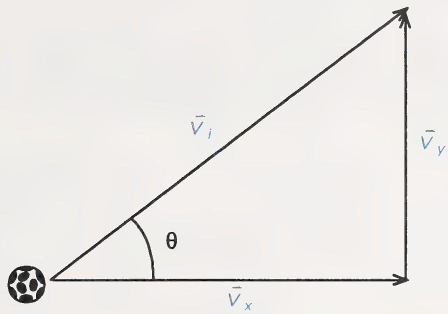
Consider the following description of a kick that scored a goal. The effects of air resistance have been ignored here.



The following sketch shows the effect of slight changes made by the kicker. Again, the effects of air resistance have been ignored.



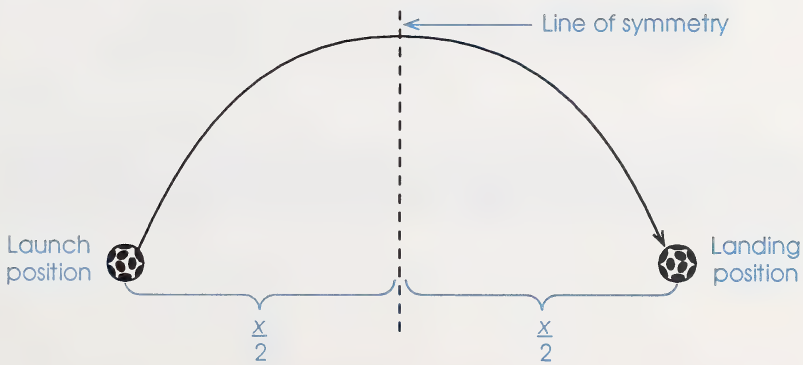
You can see that it does not take much to change the trajectory of the ball. For players, this means years of practice and good coaching. To begin analysing the motion of the ball, it is important to realize that the initial velocity can be resolved into vertical and horizontal components.



2. If air resistance is ignored, what is assumed to be true about the horizontal component of the velocity?

3. Explain the changes that occur in the vertical component of the velocity as the ball moves through its trajectory. You may find the diagram on page 137 of your textbook helpful for answering this question.

When solving projectile motion problems, you must first determine the horizontal and vertical components of the initial velocity. Assuming that air resistance is negligible, you must also realize that the trajectory of the projectile is symmetrical when the launching and landing positions are at the same height.

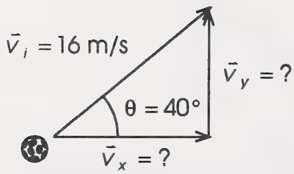


4. a. Explain why the time taken for the projectile to reach its maximum height is equal to the time taken to fall back down to the same height as the launch position.

b. How do the horizontal distances travelled in each half of the trajectory compare when air resistance is ignored?

5. A soccer ball has an initial velocity of 16 m/s and an angle of 40° to the horizontal. Analyse this motion.

a. Determine the horizontal and vertical components of v_i. In order to do this, you must fill in the missing information in the following equations.



$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

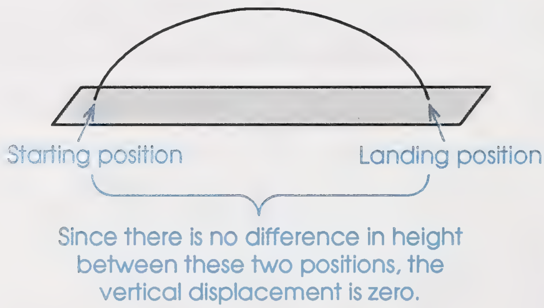
$$\cos\theta = \frac{v_x}{v_i} \qquad \sin\theta = \frac{v_y}{v_i}$$

$$v_x = \cos\theta(v_i) \qquad v_y = \sin\theta(v_i)$$

$$= (\cos\quad)(\quad) \qquad = (\sin\quad)(\quad)$$

$$= \qquad\qquad =$$

b. Determine the time in flight. The net vertical displacement for the whole trajectory is zero.



Complete the following equation to determine the answer.

$$\vec{y} = \vec{v}_y t + \frac{1}{2}\vec{g}t^2$$

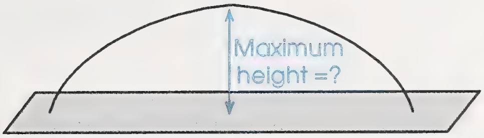
$$0 = \vec{v}_y t + \frac{1}{2}\vec{g}t^2$$

$$-\frac{1}{2}\vec{g}t^2 = \vec{v}_y t$$

$$t = \frac{-2\vec{v}_y}{\vec{g}}$$

$$=$$

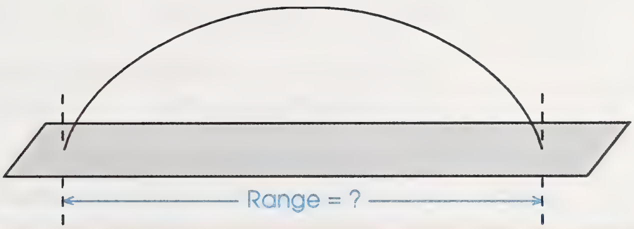
c. Determine the maximum height. The maximum height occurs at the mid-point of the trajectory. Use this information to calculate t and then substitute t into the equation.



$$\vec{y} = \vec{v}_y t + \frac{1}{2}\vec{g}t^2$$
$$= (\quad)t + \frac{1}{2}(-9.80 \text{ m/s}^2)(\quad)^2$$
$$=$$

d. Determine the range. The range occurs at the end point of the whole trajectory. Use this information to calculate t and then substitute t into the equation.

$$\vec{x} = \vec{v}_x t$$
$$= (\quad)(\quad)$$
$$=$$



Check your answers by turning to the Appendix, Section 1: Activity 4.

Another example of this type of problem can be found on page 138 of your textbook. Carefully study the strobe photo and the solution. You should not use the vector notation when finding the components of the initial velocity.

The physics of projectiles have many applications outside of the sport of soccer. To see how these concepts can be applied to football, do the following questions.

6. Do Practice Problems 5, 6, and 7 on page 139 of your textbook.

Check your answers by turning to page 667 in your textbook.

7. At what angle must a projectile be launched above the horizontal to have a maximum range? Briefly explain your answer.

When the launching height and the landing height are the same, an angle of 45° does result in the greatest range. Angles greater than 45° send the projectile vertically into the air with little horizontal motion, while angles less than 45° do not allow the projectile to stay in the air long enough to travel very far.

Real Life Applications

It is important to realize that all the calculations so far have been based on the following simplifications:

- Air resistance is negligible.
- Launch height and landing height are the same.
- Aerodynamic effects on the object in flight can be ignored.

In many cases, these simplifications are reasonable, such as when studying the jumping motion of frogs.

In other cases, these simplifications cannot be applied. When track and field athletes throw a javelin, the angle for maximum range is considerably less than 45° due to the effects of air resistance and the aerodynamics of the javelin.

In the case of a shotput, the height of the release point is about 2.0 m above the ground. This has the effect of making the optimum angle for maximum range less than 45°.

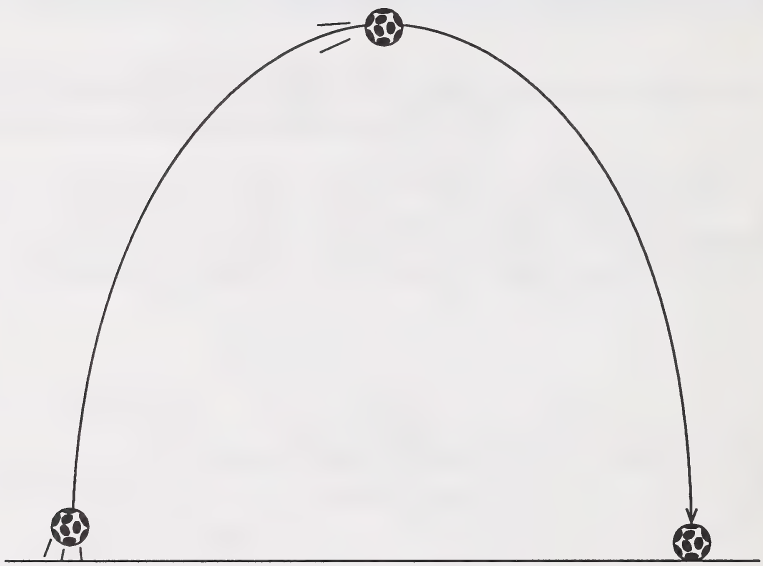
Check your answers by turning to the Appendix, Section 1: Activity 4.

Follow-up Activities

If you had difficulties understanding the concepts in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts, it is recommended that you do the Enrichment.

Extra Help

1. Summarize the key ideas about projectile motion by labelling the following diagram of a soccer ball in flight. Consider the air resistance to be zero.



   a. Label the height, the range, and the launch angle (θ).

   b. The diagram shows the position of the ball at the start, at the middle, and at the end of its path. Use a pencil to carefully draw vectors to show the horizontal and vertical components of the velocity at each position.

   c. Re-examine your vector diagrams of the horizontal velocity components from part b. How should these vectors compare to each other? Change your diagrams if necessary and explain what all these vectors should have in common if air resistance can be ignored.

d. Re-examine your vector diagrams of the vertical velocity components from part b. How should these vectors compare to each other? Change your diagrams if necessary and explain what idea links all these vectors if air resistance can be ignored.

2. Briefly state the main idea that is used to analyse projectile motion.

3. Complete the following summary chart for projectile motion.

Summary Chart for Projectile Motion		
	Horizontal Components	**Vertical Components**
Type of Motion		
Equation for Velocity Components		
Other Useful Equations		

Check your answers by turning to the Appendix, Section 1: Extra Help.

Enrichment

You should now be able to use your knowledge of projectiles to answer the following questions.

1. a. Many living animals and organisms move by jumping. When a kangaroo or frog jumps into the air, it most often leaves the ground at 45°. Explain why it is advantageous to jump at 45°.

b. Do a short research project of about 80 to 100 words outlining the sizes and shapes of legs and feet of living organisms that move by jumping. What is common to the designs of the legs and feet of such organisms?

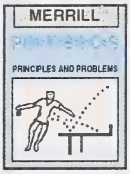
2. If you have the opportunity to work with another individual or a small group, refer to the physics lab called The Softball Throw on page 140 in your textbook. Read through the lab and follow the procedure. Set up a data table to record time and distance and show your calculations for determining the velocity of the ball.

   a. Calculate the velocity of the ball.

   b. Answer Observations and Data questions 1 and 2 on page 140 of your textbook.

   c. Answer Analysis question 1 on page 140 of your textbook.

   d. Answer Applications question 1 on page 140 of your textbook.



Science Skills

- [ ] A. Initiating
- [x] B. Collecting
- [x] C. Organizing
- [x] D. Analysing
- [x] E. Synthesizing
- [ ] F. Evaluating

Check your answers by turning to the Appendix, Section 1: Enrichment.

Conclusion

You should now realize that two independent motions are involved when projectiles are launched horizontally or at some angle above the horizontal. The two independent motions are the horizontal uniform motion and the vertical accelerated motion. If you know the initial velocity and the angle at which a projectile is launched, you can determine the maximum height and range of the projectile if air resistance is ignored.

Assignment Booklet

ASSIGNMENT

Turn to your Assignment Booklet and do the assignment for Section 1.

Section

2 Uniform Circular Motion

WESTFILE INC.

Do you enjoy carnival rides? Some people love the thrill and excitement of the high speeds. Other people feel dizzy or ill as a result of the constant acceleration.

In this section you will learn about the basic conditions that are common to all forms of circular motion. You will also develop equations for describing and explaining circular motion. By the end of this section you should be able to apply the equations to a variety of circumstances.

Activity 1: Conditions for Circular Motion

In the last section you studied the curved motion of a projectile. The path of a projectile has a parabolic shape. In this section you will explore the physics of objects that follow a circular path.

Investigation: Describing Circular Motion with Words

Science Skills

- [ ] A. Initiating
- [x] B. Collecting
- [x] C. Organizing
- [ ] D. Analysing
- [ ] E. Synthesizing
- [ ] F. Evaluating

Purpose

In this investigation you will observe circular motion and record the factors that influence the circular path.

Materials

You will need the following materials for this investigation:

- a 15-cm long glass tube that is wrapped in a cardboard sleeve or masking tape. It should be fire polished at both ends.
- 1.5 m of nylon string or 20-lb. fishing line
- a two-hole size 4 rubber stopper
- twenty metal washers with a mass of 8.0 g each
- two paper clips
- a metre stick

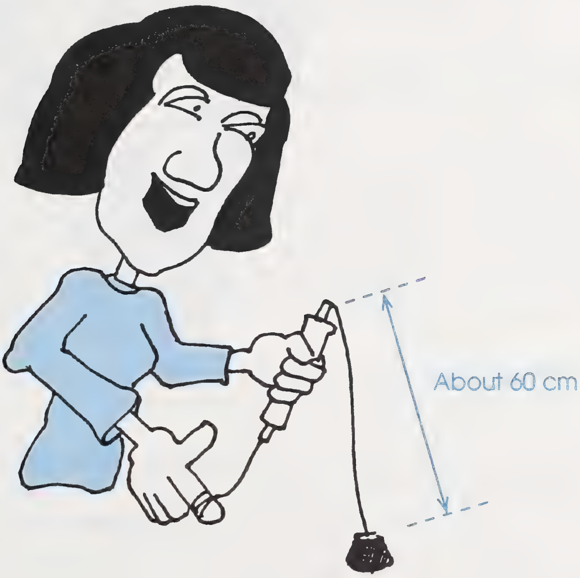
If you use washers with different masses, you will need a balance to determine the mass of each washer.

Caution

If other people are in the room with you while you are doing the lab, everyone should wear safety goggles.

Procedure, Observations, and Analysis

Securely tie one end of the nylon string to the rubber stopper. Feed the other end through the glass tubing and wrap the excess string around the index finger of your free hand. The setup of the apparatus should be similar to the setup shown in the diagram.



Use a smooth wrist motion to twirl the rubber stopper above your head. Keep the radius of the path constant in a horizontal circle. Continue twirling the stopper for 2 min.

Caution

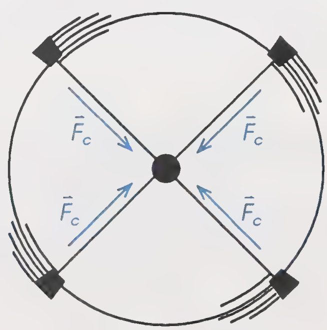
Never completely let go of the string.

1. What keeps the stopper moving in a circle? How could you test this idea?

centripetal force – the force that is directed towards the centre of circular motion

The force that your free hand provides on the string pulls the stopper towards the glass tubing. No matter what position the stopper is in, the string pulls it to the centre of its circular path. This force is called **centripetal force**. The word *centripetal* means "centre seeking".

2. Consider the top view of the apparatus. Draw instantaneous velocity vectors for the stopper on the diagram.



3. What do you notice about the direction of the instantaneous velocity and the direction of the centripetal force?

Begin twirling the stopper over your head again. Keep the radius of the stopper's path as constant as possible. Try to develop a sense of how much centripetal force you must supply to twirl the stopper at different speeds. Begin by twirling the stopper slowly and then increase the speed. Note how the centripetal force changes as the speed changes.

4. What do you notice about the amount of force that is required as the speed of rotation increases and decreases?

The next relationship to consider is the one between the centripetal force and the radius. Begin twirling the stopper in a circle over your head. Keep the radius of the path constant. Twirl the stopper at a constant speed for about 1 min. Once you have a good sense of the centripetal force that is required to maintain the circular motion, pull down hard on the string to suddenly increase the centripetal force. Note the effect on the radius of the path.

Caution

When doing the next step of this activity, be sure that there is plenty of space around you. Also be careful not to let the stopper hit you in the face.

Begin twirling the stopper over your head again, but this time suddenly reduce the centripetal force by lifting the hand that holds the string. Note the effect on the radius of the path.

5. Describe the relationship between the radius and the centripetal force.

The last part of this investigation will be done as a thought experiment because the actual procedure is too dangerous to try.

6. Imagine that you replace the rubber stopper with a much heavier object. What would you expect to happen to the centripetal force that is required? Can you think of a common experience that supports your answer?

Check your answers by turning to the Appendix, Section 2: Activity 1.

Conclusions

This investigation provided some interesting observations.

- **Increasing** the speed of rotation requires an **increase** in the centripetal force.
- **Decreasing** the radius of the path requires an **increase** in the centripetal force.
- **Increasing** the mass of the rotating object requires an **increase** in the centripetal force.

An Equation for Circular Motion

It is possible to begin developing an equation for circular motion by using the observations from the previous investigation.

Increasing the speed increases the centripetal force.	Decreasing the radius increases the centripetal force.	Increasing the mass increases the centripetal force.
$F_c = \dfrac{?v}{?}$	$F_c = \dfrac{?}{?r}$	$F_c = \dfrac{?m}{?}$

Combine all of these ideas.

$$\boxed{F_c = \frac{?mv}{?r}}$$

The trouble with this rough equation is that there is no way to tell if any of the quantities are squared, cubed, or square rooted. If you make the assumption that mass, speed, and radius are the only quantities that matter, the units for each of these quantities can be used to fine tune the equation. This process of using the units to develop the correct equation is called **dimensional analysis**.

Science Skills

- [ ] A. Initiating
- [ ] B. Collecting
- [ ] C. Organizing
- [x] D. Analysing
- [ ] E. Synthesizing
- [ ] F. Evaluating

dimensional analysis – the process of using units to help develop an equation

You can apply dimensional analysis to the equation that you just developed. You just have to remember the correct units that should be used.

- All forces are measured in newtons ($1\ \text{N} = 1\ \text{kg} \bullet \text{m/s}^2$).
- Mass is measured in kilograms (kg).
- Speed is measured in metres per second (m/s).
- Radius is measured in metres (m).

$$\boxed{F_c = \frac{?mv}{?r}} \Rightarrow \boxed{\text{kg}\left(\text{m/s}^2\right) = \frac{?\ \text{kg}(\text{m/s})}{?\ \text{m}}}$$

7. What would you have to do to the units on the right side of the equation to make it equal to the left side? (Hint: What variable has the greatest influence on the centripetal force?) If the hint doesn't help, try squaring or cubing some of the units until the equation works.

8. Use your answer from question 7 to suggest an equation for centripetal force.

Check your answers by turning to the Appendix, Section 2: Activity 1.

Activity 2: Describing Circular Motion with Numbers

In the last activity it seemed likely that the equation for centripetal force might be written as follows:

$$F_c = \frac{mv^2}{r}$$

In this activity you will collect data to confirm that this equation does accurately describe circular motion.

Investigation: Circular Motion of a Rubber Stopper

Science Skills

- [ ] A. Initiating
- [x] B. Collecting
- [ ] C. Organizing
- [x] D. Analysing
- [x] E. Synthesizing
- [ ] F. Evaluating

Purpose

In this investigation you will discover if the proposed equation for circular motion can be verified through experimentation. You will also become aware of the two methods of data analysis that will be used throughout this course.

Materials

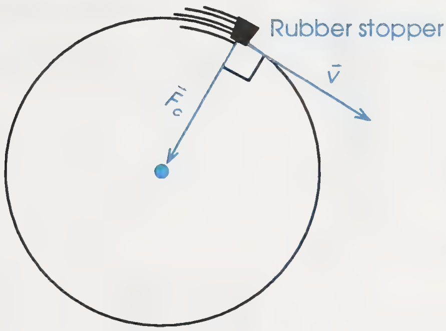
You will need the following materials for this investigation:

- a 15-cm long glass tube with a minimum outside diameter of 0.60 cm. It should be fire polished at both ends and wrapped in masking tape.
- nylon string or fishing line with a minimum length of 1.5 cm
- two-hole size 4 rubber stopper
- twenty metal washers with a mass of 8.0 g each
- two paper clips
- a stopwatch or clock with a second hand
- masking tape
- a metre stick or centimetre ruler.

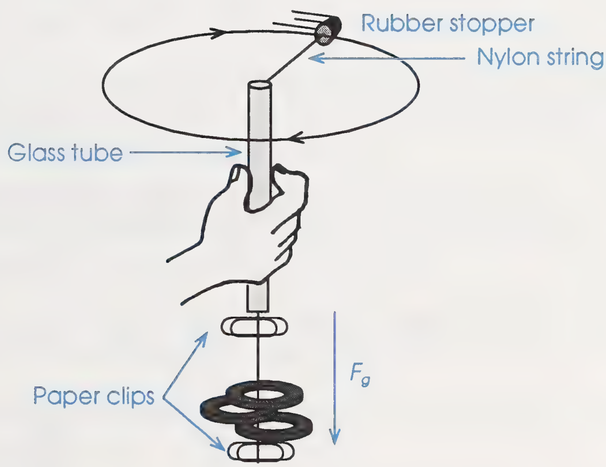
If you have access to a school laboratory, you may wish to substitute a known mass in place of the washers.

Background Information

In the last activity you learned that $\vec{v}$ is perpendicular to $\vec{F}_c$.

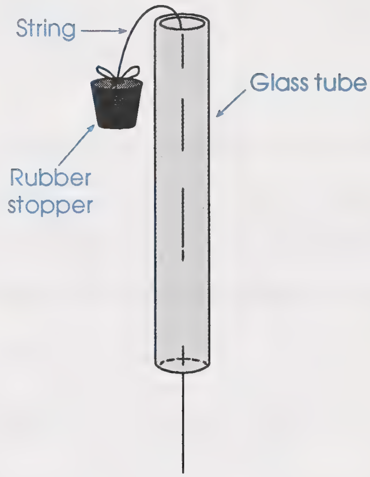


In this investigation the weight of the washers creates a tension in the string that is equivalent to the centripetal force.

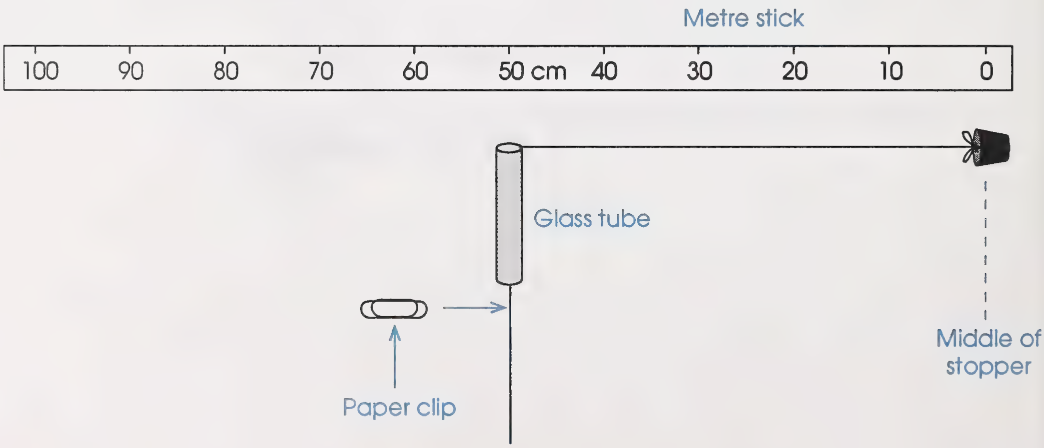


Procedure

- To avoid breaking the glass tube, wrap the tube with a layer of masking tape.
- Securely fasten one end of the nylon string to the rubber stopper.
- Pass the free end of the nylon string through the glass tube.

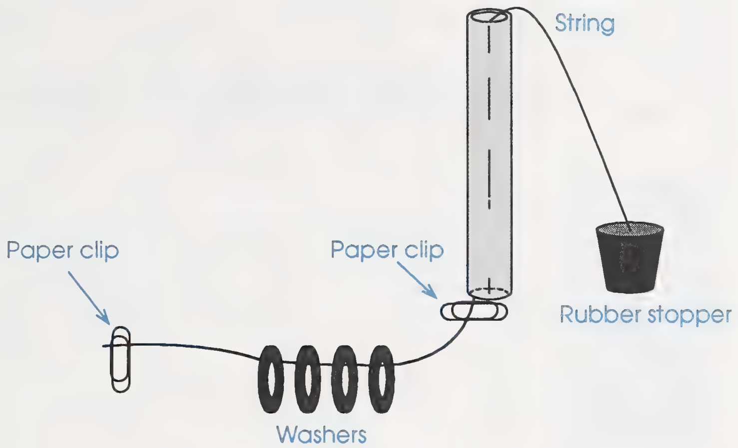


- Adjust the string so that there is 0.50 m between the top of the glass tube and the middle of the stopper.



- Fasten a paper clip to the string just below the bottom of the tube.

- Place four washers onto the free end of the string. Tie the second paper clip to the free end of the string to support the washers.



- Practise swinging the stopper in a circular horizontal path above your head before collecting any data. A smooth wrist movement will help keep the washers from moving too much.

- Adjust the speed of the stopper so that the paper clip stays slightly below the bottom of the tube. It is very important that the paper clip not be pushed tight against the glass tube.

- When you are able to keep the rotational velocity and position of the paper clip constant, record how long it takes for 30 complete revolutions. Record this value in the data chart.

- Divide the time for 30 rotations by 30 to obtain the time for one complete rotation. Record this value in the data chart.

- Add four additional washers and repeat the procedure. Record your data in the data chart.

- Repeat the procedure three more times, each time adding four more washers, until the last trial uses twenty washers.

Data

Measurements and Calculations for the Circular Motion of a Stopper							
Trial	Number of Washers	Time for 30 Revolutions (s)	Time for One Revolution (s)	Speed (m/s)	Mass of the Stopper (kg)	$\frac{mv^2}{r}$ (N)	F_c Supplied by Washers (N)
1	4						0.31
2	8						
3	12						
4	16						
5	20						

Calculations

The whole point of the investigation is to verify that $\frac{mv^2}{r}$ equals the centripetal force. In each trial the centripetal force was supplied by the weight of the washers hanging on the string. A sensible place to begin the analysis is to calculate the centripetal force for each trial.

1. Using the calculation of F_c for the first trial as an example, calculate the centripetal force for each of the other trials. Record the values in the last column of the data chart.

$$\begin{aligned}\text{Trial 1: } F_c &= W = mg \\ &= (4 \text{ washers} \times 0.008 \text{ kg/washer})9.80 \text{ m/s}^2 \\ &= 0.314 \text{ N}\end{aligned}$$

Now that you have determined the centripetal force that kept the stopper moving in the circle for each trial, you can determine the value for $\frac{mv^2}{r}$.

2. Calculate the speed of the stopper in each trial by following the calculations shown below. Record your values for speed on the data chart.

$$v = \frac{\text{distance travelled}}{\text{time taken}} = \frac{\text{circumference of the path}}{\text{time to travel one revolution}}$$
$$= \frac{2\pi r}{T}$$

3. Use the mass of the stopper, the speed, and the radius to calculate the value of $\frac{mv^2}{r}$ for each trial. Record these values on the data chart.

Analysis

Does $\frac{mv^2}{r}$ equal F_c? The investigation now comes down to answering this one question. If you examine the data closely, some other questions will be brought to mind.

- How close do the values of the numbers have to be in order to be considered equal?
- Since these numbers are based on measurements, and measurements always include errors, what percent error are you willing to accept for this investigation? Would 20 percent error allow you to conclude that $F_c = \frac{mv^2}{r}$? Or would you only accept ten percent error?
- What factors contributed to measurement errors in this investigation?

Two approaches are presented to help you analyse the last two columns of the data table. Complete both methods.

Approach 1: Data Averaging

4. Consider the value for F_c to be the theoretical or accepted value for the centripetal force. Consider the value of $\frac{mv^2}{r}$ to be the experimental value for the centripetal force. Use these numbers to calculate the percent error for each trial. Write your answers in the following chart.

Percent Error for Each Trial			
Trial	Experimental Value (N)	Theoretical Value (N)	Percent Error $= \frac{\lvert \text{Experimental} - \text{Theoretical} \rvert}{\text{Theoretical}} \times 100\%$
1			
2			
3			
4			
5			

5. Obtain an average value for the percent error for the whole experiment.

6. Condsidering the equipment and techniques used in this experiment and the errors that occurred in measuring, can you say that your percent error is acceptable? Does $\frac{mv^2}{r}$ equal F_c within experimental error?

Approach 2: Graphical Analysis

In this experiment the mass of the stopper and the radius of the path were kept constant. The two variables were the velocity and the centripetal force supplied by the washers. Instead of graphing v and F_c, you will plot v^2 and F_c. Although the force is actually the manipulated variable, you will plot it on the vertical axis for this analysis.

7. Complete the following chart to prepare for graphing.

Data for Graph			
Trial	v (m/s)	v^2 (m^2/s^2)	F_c Supplied by Washers (N)
1			
2			
3			
4			
5			

8. Plot the data from the chart in question 7 on the graph paper on the next page. Draw the best fit line.


Centripetal Force
(N)
Speed Squared
(m²/s²)

9. Find the slope of the best fit line.

10. What does the slope represent in terms of the physics of this investigation?

11. Calculate the percent error with your answers to questions 9 and 10.

12. Considering the equipment and the techniques used in this experiment and the errors that occurred in measuring, can you say that your percent error is acceptable?

Interpretation

13. Compare the two methods of data analysis. What are the advantages and disadvantages of the graphical analysis method?

14. Do you think that it is possible for the two methods of data analysis to produce different values for the percent error? Explain your answer.

Conclusions

15. Did you confirm that the equation $F_c = \frac{mv^2}{r}$ is valid? Explain your answer by accounting for your percent error.

In this activity you have increased your understanding of the conditions that describe circular motion. Although the equipment that you used was very simple, you should have developed more confidence in the equation $F_c = \frac{mv^2}{r}$. When experiments are completed with more sophisticated apparatus, this equation does actually describe uniform circular motion. In the next activity you will see how this equation can be applied to a number of different circumstances.

Check your answers by turning to the Appendix, Section 2: Activity 2.

Activity 3: Going in Circles: Applications

Before you begin to investigate how uniform circular motion can be applied to common circumstances, you should review the main ideas from the last two investigations.

- All objects moving uniformly in a circle require a centripetal force. The centripetal force is always directed to the centre of the circular path.
- The two equations for circular motion and their units are shown.

Mass of the object moving in the circle

Centripetal force

$$\boxed{F_c = \frac{mv^2}{r}}$$

Speed squared of the object moving in the circle

Radius of the circular path

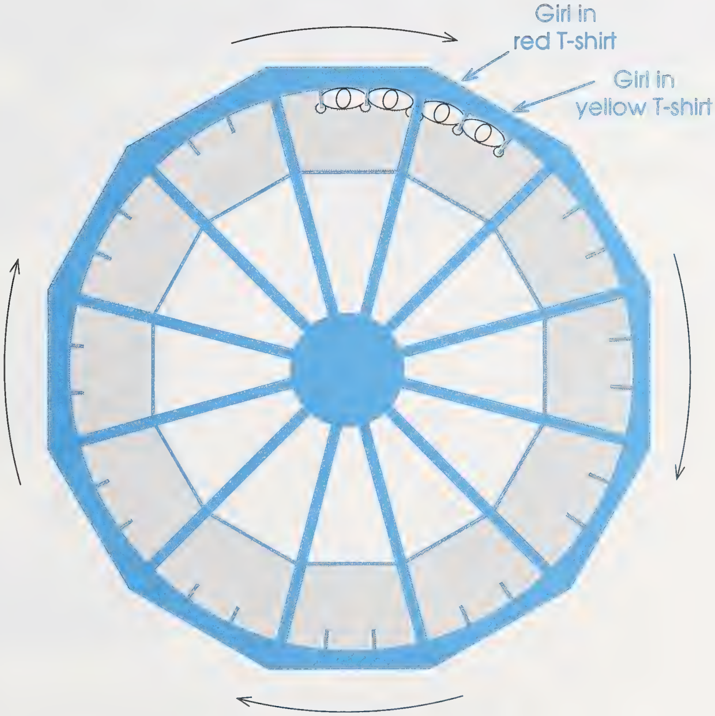
Speed of the object moving in the circle

$$\boxed{v = \frac{2\pi r}{T}}$$

Radius of the circular path

Period of time for one revolution

One of the most exciting applications of the physics of circular motion occurs on the rides at amusement parks. Turn to page 142 in your textbook and look at the picture of four people on an amusement ride. A top view of this carnival ride is shown in the following sketch. Find the girls in the yellow and red T-shirts in both the photograph and the sketch.



1. Draw a vector on the previous sketch to show the centripetal force acting on the girl in the yellow T-shirt.

2. Draw a vector on the previous diagram to show the velocity of the girl in the yellow T-shirt.

3. Consider the following sample data for the girl in the yellow T-shirt.

   mass = 52.5 kg
   distance from the centre of the ride = 4.1 m
   time for 2 revolutions = 6.2 s

   Use the sample data to calculate the centripetal force acting on the girl.

There is another method for calculating the centripetal force. This method combines two steps into one. This method creates a new equation.

$$\boxed{v = \frac{2\pi r}{T}}$$

Substitution

$$\boxed{F_c = \frac{mv^2}{r}} \rightarrow \boxed{F_c = \frac{m\left(\frac{2\pi r}{T}\right)^2}{r}}$$

Square the speed.

$$\boxed{F_c = \frac{m4\pi^2 r^2}{T^2 r}}$$

Cancel *r*.

$$\boxed{F_c = \frac{m4\pi^2 r}{T^2}}$$

Many students find this equation helpful because it allows a two-step problem to be done in one step. Just be sure that you know what the variables represent.

Mass of object moving in the circle

Centripetal force

$$\boxed{F_c = \frac{m4\pi^2 r}{T^2}}$$

Radius of the path

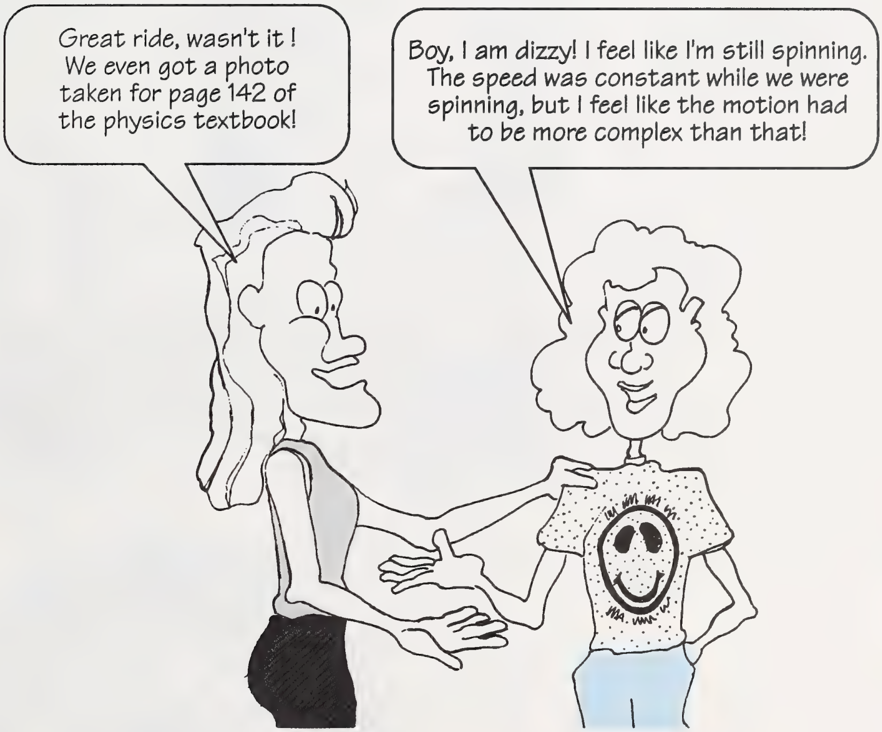
Period (time for 1 revolution)

4. Re-attempt question 3 using the new equation.

5. What supplies the centripetal force that keeps the girl moving in the circular path?

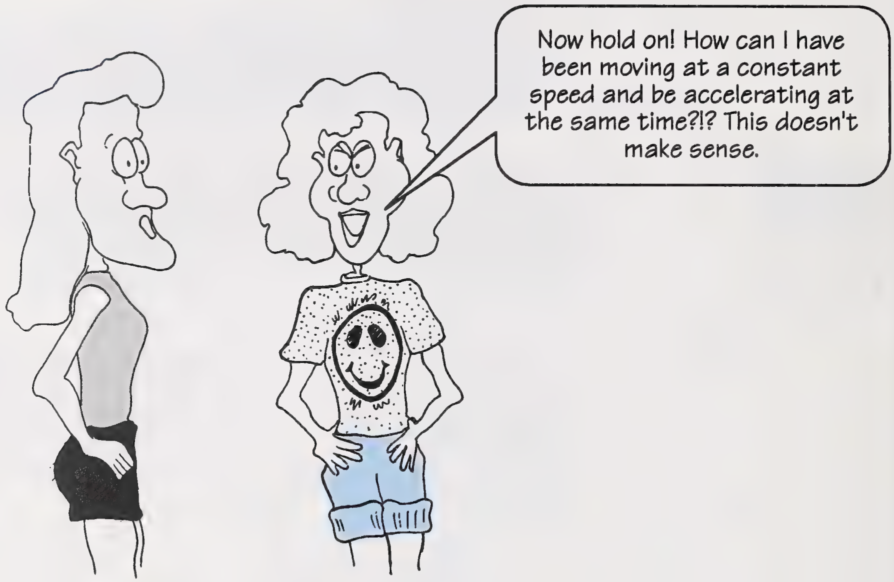
Check your answers by turning to the Appendix, Section 2: Activity 3.

To deepen your understanding of circular motion, it might be helpful to hear the reactions of people who have actually been moving in circles. Imagine a conversation between the girls who were featured in the photograph on page 142 of your textbook.

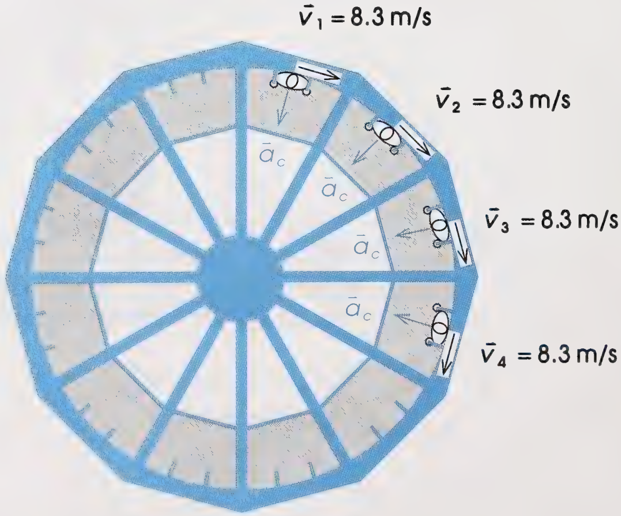


An important point is being raised – there is more to this motion than constant speed. Newton's second law helps explain this concept.

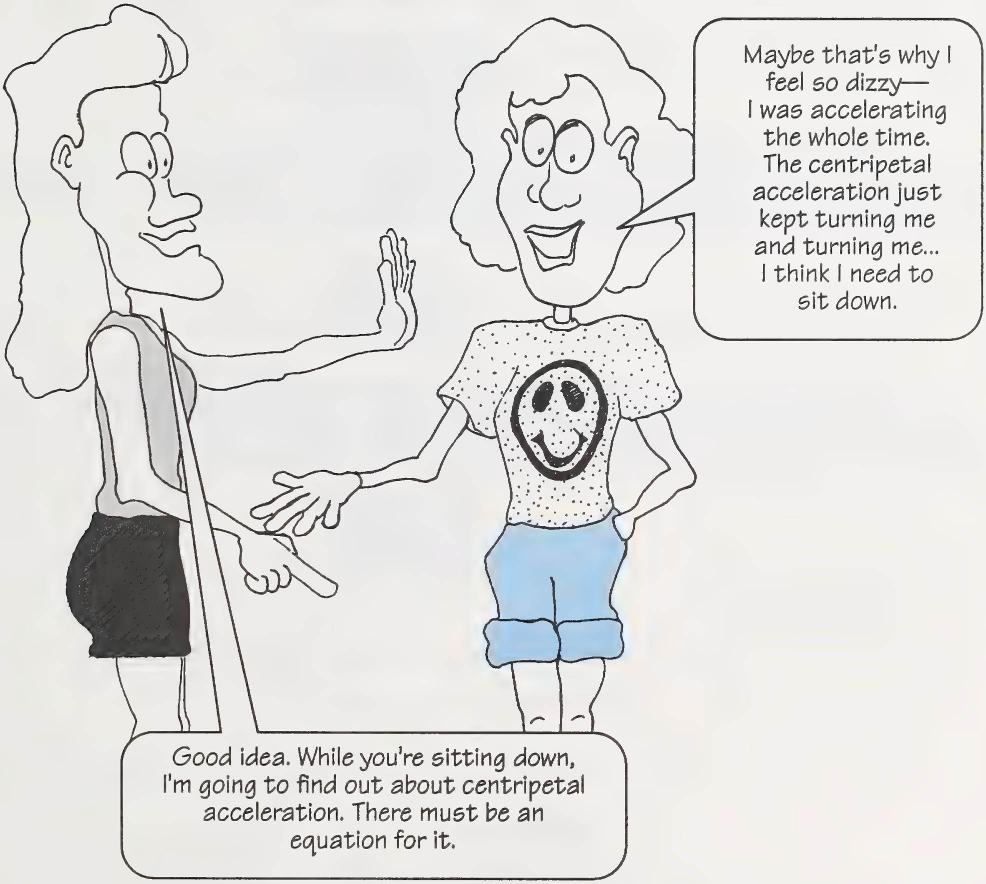
All net forces cause objects to accelerate in the direction of the force. In this case, the net force is a centripetal force. Newton's second law requires that there should be a centripetal acceleration directed to the centre of the circle.



Actually, this does make sense. The confusion is with the words *speed* and *velocity*. While the speed was constant, the velocity was continually changing. In this case, the acceleration did not change the **magnitude** of the velocity (the **speed**), but the acceleration changed the **direction** of the velocity. The following diagram shows the girl in the yellow shirt at different positions on the carnival ride.



The centripetal acceleration is always directed towards the centre of the circle since this is the direction of the centripetal force. This means that the centripetal acceleration slightly changes the direction of $\bar{v}_1$ to the direction of $\bar{v}_2$, and then changes the direction of $\bar{v}_2$ to the direction of $\bar{v}_3$, and so on.



You will be glad to know that the equations for centripetal acceleration follow directly from Newton's second law.

$$\boxed{F_{net} = F_c}$$

The centripetal force is the net force.

$$\downarrow$$

Substitution

$$\boxed{ma = \frac{mv^2}{r}}$$

$$\downarrow$$

Cancel the mass from both sides.

$$\boxed{a = \frac{v^2}{r}}$$

$$\downarrow$$

The acceleration is a centripetal acceleration since it is always directed to the centre.

$$\boxed{a_c = \frac{v^2}{r}}$$

6. Another equation for centripetal acceleration is $a_c = \frac{4\pi^2 r}{T^2}$. Show two different ways to derive this equation.

7. Use the data for the girl and the carnival ride to answer this question.

   mass = 52.5 kg
   radius = 4.1 m
   period = 3.1 s

   a. Use the equation $a_c = \frac{v^2}{r}$ to determine the centripetal acceleration of the girl.

   b. Use the equation $a_c = \frac{4\pi^2 r}{T^2}$ to determine the centripetal acceleration of the girl.

c. Use your earlier answers for centripetal force and Newton's second law to calculate the centripetal acceleration of the girl.

d. Compare the value for centripetal acceleration to the acceleration due to gravity.

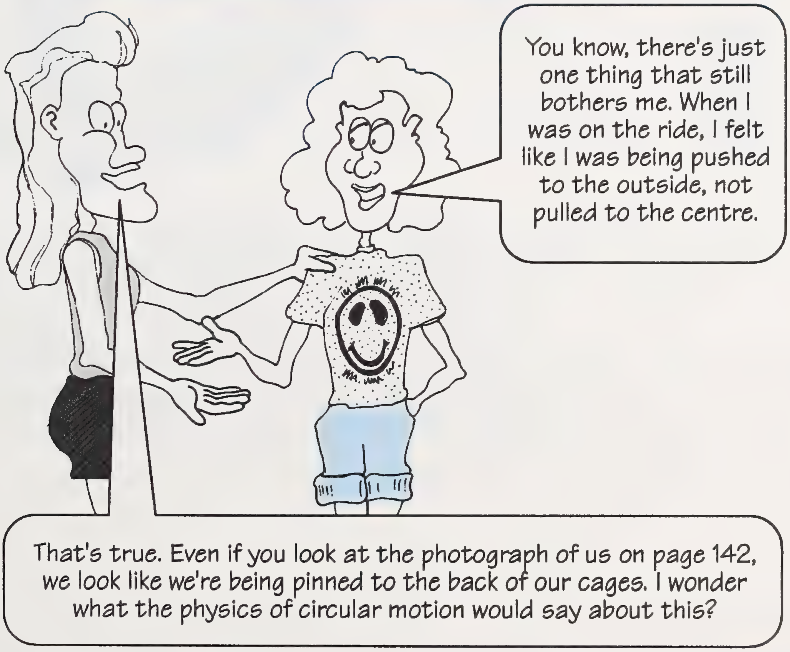
Carefully read from the bottom of page 141 through to the Example Problem on page 145 in your textbook. Read carefully because the textbook uses a slightly different treatment of vector notation.

8. What are the differences between the equations that were presented in this module and those presented in the textbook?

Check your answers by turning to the Appendix, Section 2: Activity 3.

9. Do Practice Problems 10 and 11 on page 145 of your textbook. Be sure to use the scalar versions of the equations presented in this module.

Check your answers by turning to page 668 in your textbook.



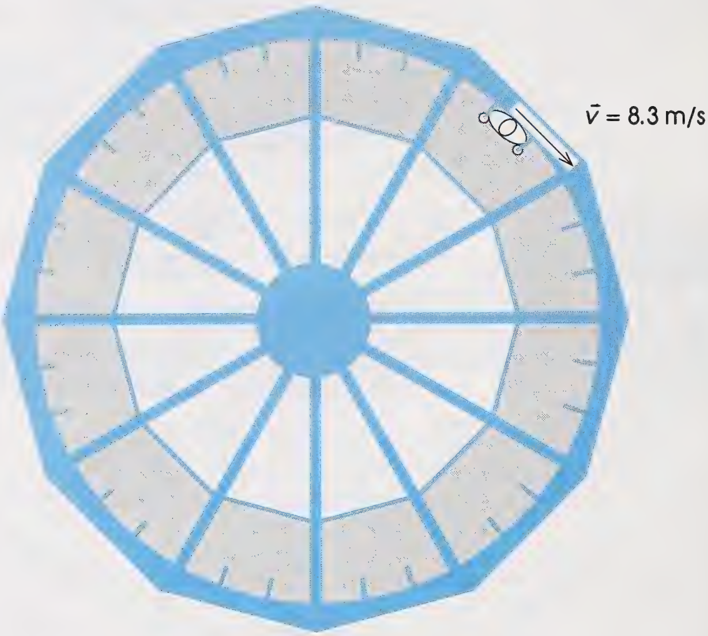
lab frame of reference – The observer is attached to the ground, or the lab, and is not attached to what is observed to be moving.

rotating frame of reference – The observer is attached to the rotating object and makes observations from this point of view.

The best way to answer this question is to recognize that there are two ways to look at this situation. One point of view is called the lab frame of reference and the other is called the rotating frame of reference.

Lab Frame of Reference

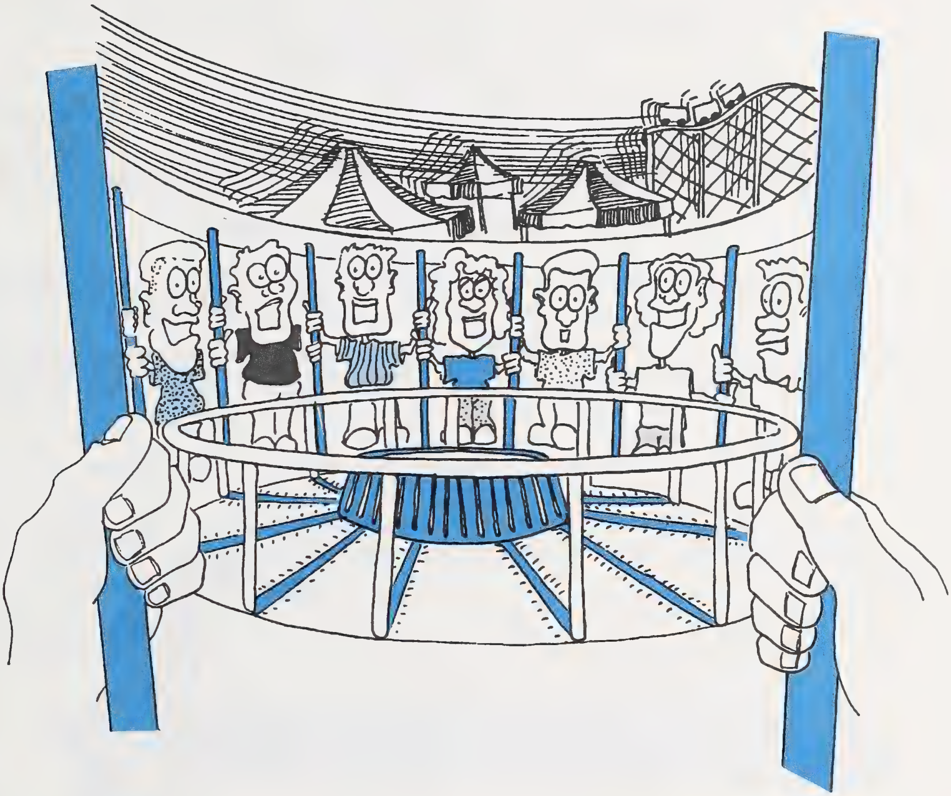
From this point of view, you are attached to the ground and are not moving with the carnival ride. Imagine yourself looking down on the amusement ride. You are still looking at the girl in the yellow shirt.



Newton's first law states that objects tend to maintain their velocity unless acted on by a net force. Luckily, the girl does not fly off the rotating ride! The cage exerts a centripetal force which causes her to change direction. The cage, in turn, is held in place by the steel pipes and rods that attach it to the middle of the ride. This ensures that the cage does not fly off the ride. The steel pipes exert a force on the cage, which exerts a force on the girl. The girl feels this force as a pressure on her back.

Rotating Frame of Reference

If you use this point of view, you must imagine yourself being on the amusement ride.



As you spin, you feel the pressure from the cage on your back. You also notice that your arms feel pinned to the back of the cage. From this frame of reference, the world outside the ride is spinning rapidly around you.

centrifugal force – a ficticious force used by an observer in a rotating reference frame to explain observations

The force that you think is pushing you against your cage is called a centrifugal force. This force is called a ficticious force because it is noticeable only in the rotating frame of reference. The reason why you actually feel that your arms are forced to the back of the cage is because they have a natural tendency to move in a straight line like the rest of your body, but the cage is getting in the way.

When you solve problems in this course, it is the lab frame of reference that you will use. The centripetal force is the one you will use in your calculations. It is important not to mix up centripetal and centrifugal forces and the two frames of reference.

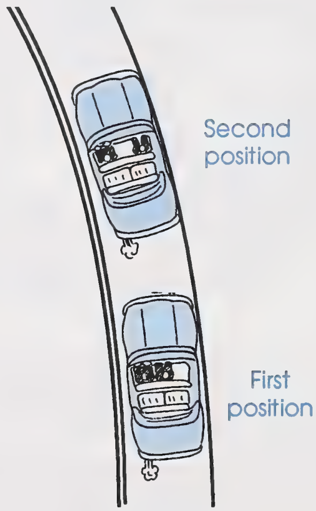
An Everyday Application

Imagine that you are a passenger in a convertible that has vinyl seats. You are sitting in the middle seat, right next to the driver. Suddenly the driver turns a sharp corner and you slide across the seat to the passenger door.

10. What is your frame of reference?

11. Which force would you use to explain your observations?

Imagine that a helicopter was hovering in one spot above the turn that the convertible just made. A view of the car in several positions is shown.



12. Write the following labels on the previous diagram.

    a. Indicate the direction of the centripetal force on the driver in both positions.

    b. Draw an arrow from your first position as a passenger to your second position.

    c. Label the helicopter's frame of reference on the diagram.

13. Consider the arrow that you drew from your first position to your second position to be the velocity in position 1. How does Newton's first law explain your slide across the seat towards the door?

14. Are you really being "forced" into the door? Explain your answer from the lab frame of reference.

15. What provides the centripetal force to keep you and the car moving in a curve?

16. What could you do to avoid sliding across the seat?

The physics of circular motion applies to many topics other than carnival rides and automobiles. Discover some of these applications as you work through the next problems.

17. Turn to page 153 of your textbook and do Problems 14, 16, 19, and 20.

Check your answers by turning to the Appendix, Section 2: Activity 3.

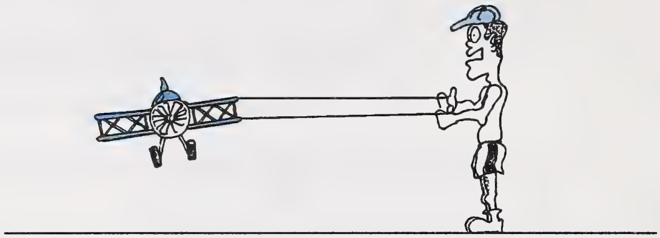
Follow-up Activities

If you had difficulties understanding the concepts in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts, it is recommended that you do the Enrichment.

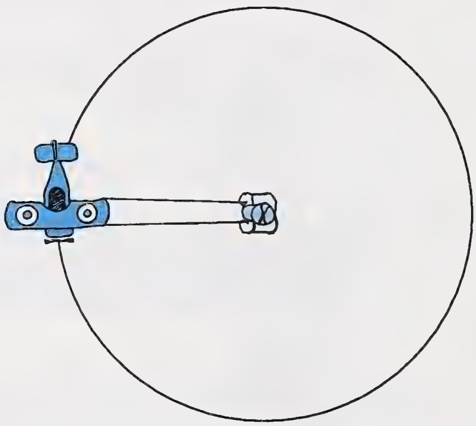
Extra Help

A popular hobby is building gas-powered model airplanes and flying them by remote control. The remote control may take the form of a radio signal or the plane may be attached to long lines that the hobbyist holds in the centre of the plane's circular path.

Side view:



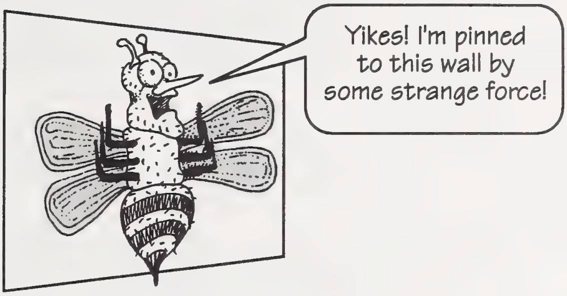
Top view:



1. Draw a vector to show the direction of the centripetal force on the side view of the model airplane.

2. Draw vectors to show the centripetal acceleration and the velocity of the airplane on the top view of the model airplane.

3. Imagine that a hornet accidently found its way into the tiny cockpit of the model airplane prior to take-off. Explain the hornet's observations from his frame of reference and from the lab frame of reference.



4. The airplane has a constant speed, but it is also continually accelerating. Use Newton's laws to explain these facts.

5. The model airplane has a mass of 2.04 kg, travels at 19.1 m/s, and moves in a radius of 9.1 m. Determine how long the airplane takes to make one circle and how much force is required to keep it moving in a circle.

Check your answers by turning to the Appendix, Section 2: Extra Help.

Enrichment

Do **one** of the following activities.

1. Do the following investigation.

Investigation: Plants in Circular Motion

Science Skills

- [ ] A. Initiating
- [x] B. Collecting
- [x] C. Organizing
- [x] D. Analysing
- [x] E. Synthesizing
- [ ] F. Evaluating

Purpose

In this activity you will investigate the effects of circular motion on the direction of shoot and root growth from bean seeds.

Materials

You will need the following materials for this investigation:

- an old record player with a 78 r/min speed
- three identical small glass jars
- six bean seeds
- paper towel
- epoxy glue
- an old long-play phonograph record
- a large cardboard box

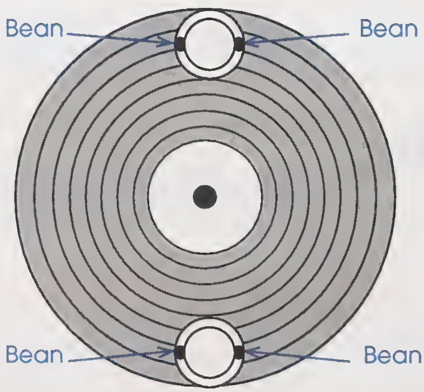
The record and record player may be ruined by the end of this experiment. You will need to find an old record and record player for this investigation.

Procedure

- Fill the jars with paper towel.

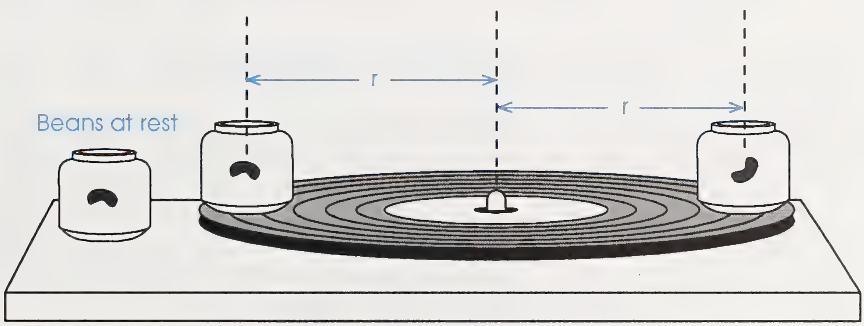
- Place the bean seeds on opposite sides of the jar. Add enough water to soak the paper towel.

- Carefully glue the jars onto the record with the epoxy. Be sure to align the jars so that they are exactly opposite each other on the record and so that the beans are positioned as shown.



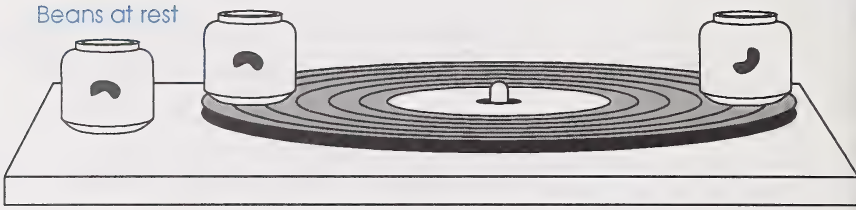
- When the epoxy glue has dried, place the record on the record player. Place the third jar next to the record player.
- Turn the record player on and adjust the speed to 78 r/min.
- Place the large box over the whole apparatus. Be sure to allow air to enter and leave the box so that the record player does not overheat.
- Wait two weeks. You will need to add water to the jars every few days. This will also give you a chance to check the growth of the beans.
- While you are waiting, research the topic of geotropism in a biology textbook. Note that the bean seed is in a rotating reference frame where an imaginary centrifugal force could be observed.

a. Predict which way the roots and stems of the beans will grow. Sketch your predictions on the following diagram.



Observations

b. After two weeks, sketch the way that the bean stems and roots grew on the following diagram.

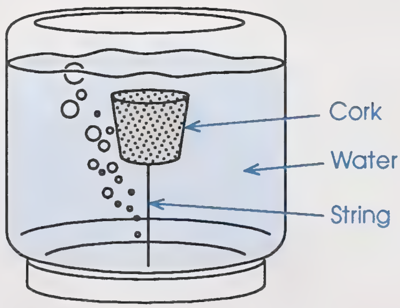


Conclusions

c. What is the effect of the rotating frame of reference on the stems and roots? Use what you learned about geotropism to answer.

2. **The Jelly Jar Accelerometer**

To demonstrate that an object moving in a circle at a constant speed is accelerating, construct a simple accelerometer. Obtain a 1-L jar and attach a string to the centre of the lid by using waterproof tape or by punching a small hole in the lid and running the string through the hole. After the string has been inserted, the hole can be sealed with putty. Tie a cork onto the other end of the string, fill the jar with water, screw on the lid, and turn the jar upside down.



The displacement of the cork always points in the direction of the acceleration. Hold the jar out in front of you and turn around at a steady speed. Observe the cork while turning around and explain your results.

3. Observing People on a Carnival Ride

Science Skills

- [ ] A. Initiating
- [x] B. Collecting
- [x] C. Organizing
- [x] D. Analysing
- [ ] E. Synthesizing
- [ ] F. Evaluating

This activity requires you have to have access to the laser videodisc entitled *Physics: Cinema Classics* and a laser videodisc player with a bar code reader. If your laser videodisc player does not have a bar code reader, enter the frame numbers provided with the icon to search and play each sequence.

Load Side B of the laser videodisc into the laser videodisc player and press "play" to spin the disc. Use the bar code reader to watch the people spinning from the lab frame of reference.

Frames 12887 – 13373

a. What information would you need to determine the centripetal acceleration of the people on the ride?

b. Use the bar code reader to collect some of the essential data.

Frame 12888

c. Use the stop function capabilities and the bar code reader to collect data from the sequence so that you can calculate the period of rotation.

Frames 12887 – 13373

d. Calculate the centripetal acceleration.

Use the bar code reader to watch the effect that the rotating rider has on a ball that is hung from a rope.

Frames 13374 – 13554

e. What frame of reference is used for observing the ball on the rope?

f. What force caused the ball to swing to the outside of the ride? Answer from the point of view of the person holding the rope.

Check your answers by turning to the Appendix, Section 2: Enrichment.

Conclusion

You have now examined the motion of objects travelling a constant speed along a circular path. The centripetal acceleration experienced by the object is the result of a centripetal force which, like the acceleration, is directed towards the centre of the circular path. The centripetal force described by Newton's second law is often the result of friction or tension in a string attached to a rotating object. However, there are many other ways to provide a centripetal force.

In the next section you will investigate the motion that results when gravity supplies a centripetal force.

Assignment Booklet

ASSIGNMENT

Turn to your Assignment Booklet and do the assignment for Section 2.

Section

3 Gravitation

NASA

Earth is often compared to a huge spaceship travelling at terrific speeds through space. It is amazing that such a large object that is travelling so fast can stay in its orbit. Why doesn't Earth fly off into space? How would you answer this question?

Many people would use the word *gravity* in their answer, but it is likely that they really do not know what the word means. This is not surprising, since some of the greatest minds in the history of physics have admitted that they are unsure about the origins of gravity. Even Isaac Newton, whose description of gravity revolutionized the way people thought of the universe, could not explain the mechanisms that cause gravity to occur.

In this section you will consider the history of observing motion in the heavens. You will also investigate the work of Kepler and see how it blends with Newton's law of universal gravitation. The section ends with an examination of satellite motion that will help you review the key ideas from Module 3.

Activity 1: Things Are Looking Up

If you have ever spent a night in the wilderness, far from the lights of a town or village, you know how overwhelming it is when you look up into the sky and see the incredible number of stars. People have been looking into the skies for thousands of years; partly because they were just as star struck as you were, and also because their existence depended on it.

1. Imagine that it's 2000 years ago and you are a fisherman in an open boat on a large body of water. It is a dark, moonless night and the stars are your only companions. How could the stars help you find your way home?

In some ways you are not unlike that ancient fisherman because your life still depends on a star – the sun.

2. Why does your existence depend on the sun?

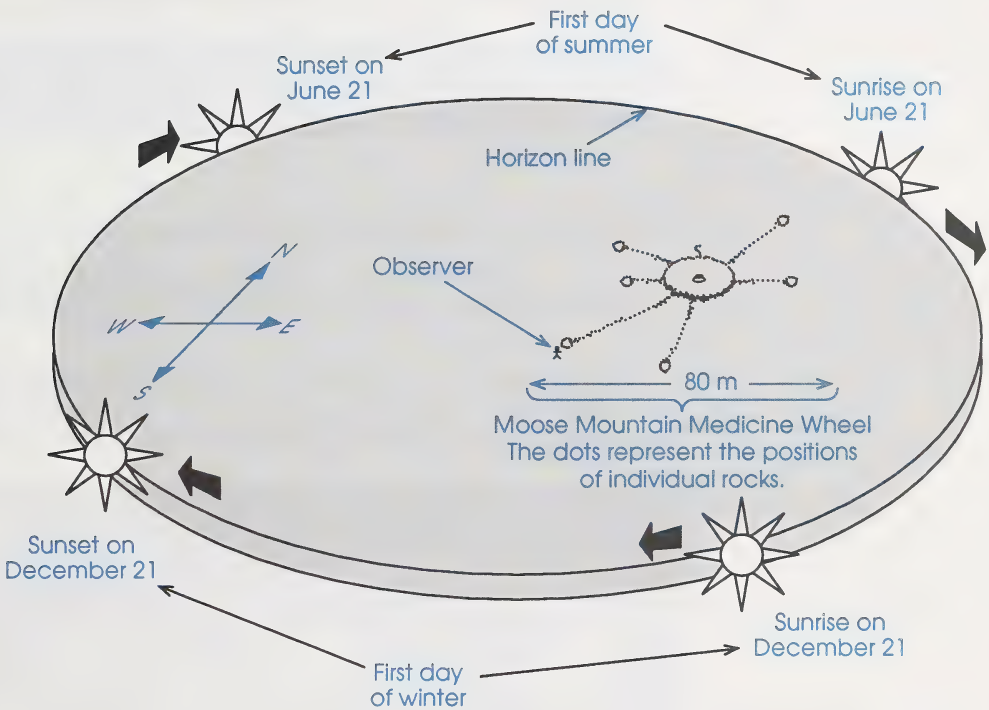
The sun, and the motion of the earth around it, has a profound influence on almost every aspect of your life. The weather, your region's climate, and the passing of the seasons are all governed by how energy from the sun interacts with the earth's surface characteristics and the motion of the planet.

For ancient peoples, knowing the passing of the seasons was a matter of life and death. The migration of buffalo, the harvesting of berries, and the planting of crops all depended on being able to carefully track the progress of the seasons.

The changing of the seasons could be anticipated by building devices that allowed careful observation of the position of the sun at sunrise and sunset. These devices often took the form of large circular arrangements of rocks or buildings that were constructed on hilltops. By standing in special positions, an observer could align the rays of the rising sun with particular openings within the rocks. These structures could also be used for locating the position of important constellations of stars along the horizon.

Some archeologists suggest that this was the purpose of the Stonehenge site in England. This explanation has also been suggested for the large circular arrangement of boulders that is found on the Canadian prairies. These sites, known as medicine wheels because they resemble spoked wheels, were created by the ancestors of native peoples. The Moose Mountain Medicine Wheel in Saskatchewan is thought to have been built around 600 B.C. Although the exact meaning of this medicine wheel remains a mystery, it is believed that its purpose was to act like a gigantic calendar. If this is true, it is the oldest astronomical observatory in North America.

3. You can demonstrate how the Moose Mountain Medicine Wheel may have worked by using a ruler to draw a line from the centre of the sun rising on June 21 to the observer. Carefully draw this line on the following diagram.



4. If the purpose of the Moose Mountain Medicine Wheel was to make astronomical observations, why was it made so large?

There is similar evidence from all over the world to suggest that ancient peoples made astronomical observations. Many people think that the very first topic studied in physics was probably **astronomy**.

astronomy – the study of the brightness, motion, and composition of objects in the heavens

As you learn how to apply Newton's laws and the physics of curved motion to the study of objects in the heavens, keep in mind that this science did not begin with Isaac Newton. For thousands of years, physics was a matter of life and death.

5. Why was astronomy so important for ancient peoples?

Check your answers by turning to the Appendix, Section 3: Activity 1.

Activity 2: Kepler Describes Planetary Motion

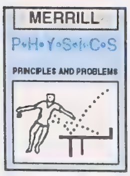
You have probably observed the moon at different positions in the sky. If you have had access to a telescope, you may even have observed planets. The photo shows a view of Saturn from the Hubble Space Telescope.

NASA

celestial bodies – bodies in the sky or visible heavens

astronomer – a scientist who studies astronomy

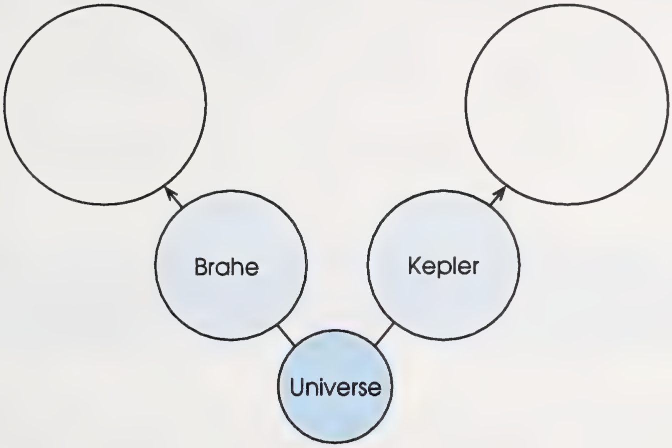
The positions of these **celestial bodies** change over time. It was these changes that inspired an **astronomer** named Tycho Brahe to record the exact location of the planets and stars in the mid 1500s.



Read about Tycho Brahe's story and his introduction to Johannes Kepler on page 156 in your textbook.

1. What event prompted Brahe to dedicate his life to the careful study of astronomical events?

2. Kepler and Brahe had different ideas about how the universe worked. Complete the following chart that contrasts their ideas.



Now continue reading on page 157 to discover Kepler's contribution. Pay special attention to the list of Kepler's three laws.

3. Kepler devised both the mathematics and the theory to explain Brahe's very precise measurements. Which of these is no longer considered to be correct?

MERRILL P•H•Y•S•I•C•S PRINCIPLES AND PROBLEMS

ellipse – a closed plane curve formed by the elongation of a circle

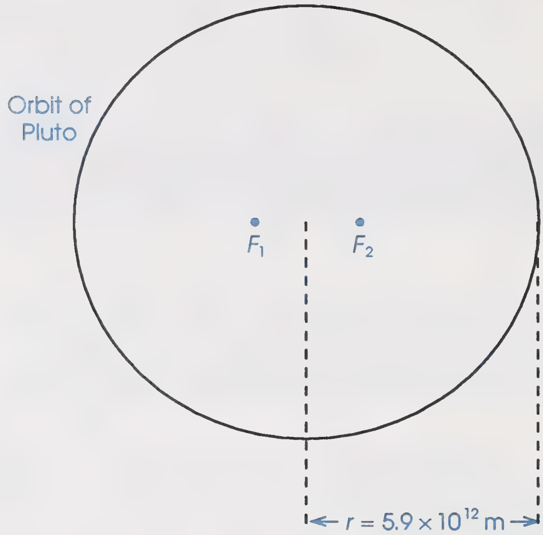
focus – one of two fixed points used in the creation of an ellipse

The planets revolve around the sun in orbits that are shaped like an ellipse. The best way to understand ellipses is to learn how to make one. Read the procedure and study the diagram on page 158 of your textbook.

4. How could you modify the procedure to make a circle instead of an ellipse? Refer to the position of each focus in your answer.

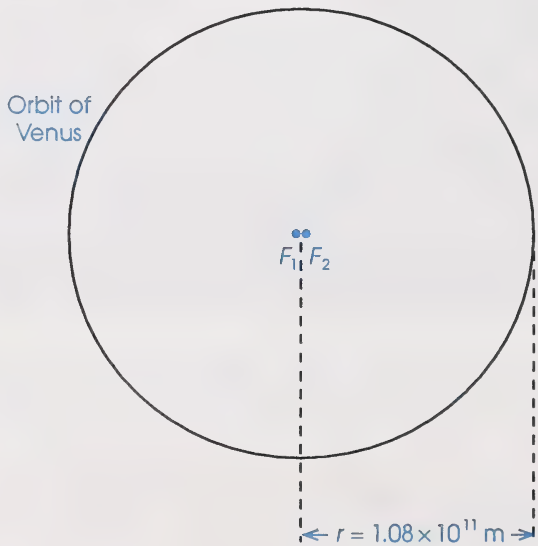
5. Why is a circle sometimes regarded as being a special case of an ellipse?

The orbits of the different planets vary. The planet Pluto has the most elliptical orbit of all, as shown in the following diagram.



6. If the sun is considered to be at the focus labelled F_1, why is it more correct to call r the mean distance from the sun rather than the radius?

The other planets have orbits that are more circular than Pluto's. Venus has the most circular orbit of all the planets.



7. If the sun is considered to be at the focus labelled F_1, why is the mean distance from the sun (r) very close to actually being the radius of the orbit?

The other planets have orbital shapes that fall within the range set by Venus and Pluto. Most are closer to the circular shape of the path of Venus than they are to the oval shape of the path of Pluto.

8. Carefully examine Figure 8-2 on page 157 of your textbook.

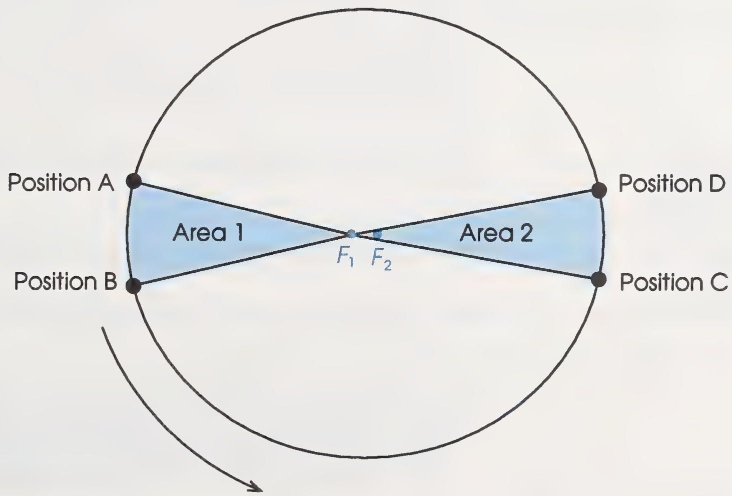
    a. Why could this figure be considered misleading?

    b. Why do you think that it was drawn this way?

Check your answers by turning to the Appendix, Section 3: Activity 2.

Kepler developed three laws of planetary motion. The first law states that planets have an elliptical orbit with the sun at one focus.

Kepler's second law can be illustrated by the following diagram which shows a planet in four different positions. The sun is considered to be at the focus labelled F_1.



9. If Area 1 is equal to 1.72×10^8 km^2, what is Area 2 equal to?

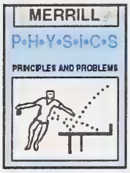
10. Between which two positions would the planet have the greatest velocity? Explain your answer.

To become familiar with Kepler's third law, answer the following questions.

11. a. In the previous section, the term *period* was introduced. Describe this term as it applies to a planet moving around the sun.

    b. What is the period of Earth revolving around the sun?

Kepler's third law is best understood by solving problems. It is important to have a clear understanding of how your calculator handles things like squares, square roots, cubes, and cube roots.



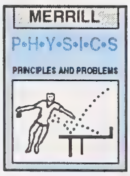
Carefully read the Problem Solving Strategy outlined on pages 157 and 159 in your textbook. Study the Example Problems on pages 159 and 160 and try the calculations with your calculator to be sure that you know how to use it properly. If you run into problems, consult the instruction booklet for your calculator and keep trying the calculations.

Questions 12 to 14 refer to the Example Problem on page 159 of your textbook.

12. What is located at one focus for the two moons in the Example Problem?

13. Which units are used to measure the period for Jupiter's moon Ganymede?

14. Why are the units for the radius not significant in determining the period?



Questions 15 and 16 refer to the Example Problem on page 160 of your textbook.

15. What is located at one focus for the moon's orbit in this example?

16. What happens to the units for the period as you go through the calculation?

Check your answers by turning to the Appendix, Section 3: Activity 2.

17. Do Practice Problems 1, 2, 3, and 4 on page 160 of your textbook.

Check your answers by turning to page 668 in your textbook.

In the next activity you will examine the force which is responsible for maintaining planets within a prescribed path.

Activity 3: Newton and Universal Gravitation

Newton's law of universal gravitation – the law which describes the attraction between two masses

Newton's law of universal gravitation is said to have developed after Newton observed an apple falling from a tree. He was trying to describe the nature of the force that accelerates the apple or any other body towards the earth. Newton also wanted to describe the nature of the force exerted by the sun that provides a centripetal acceleration on the planets to keep them in orbit. What clues were available to Newton about the nature of the force?

Read pages 160 and 161 of your textbook. Then answer the following questions.

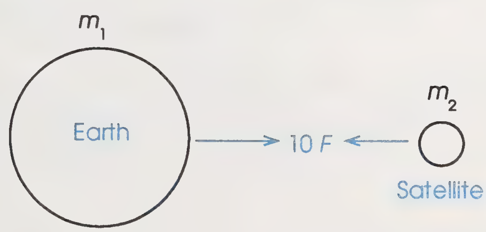
1. What information about the two objects attracting each other is essential to Newton's ideas about the force of gravity?

2. Write a proportion relating force and distance.

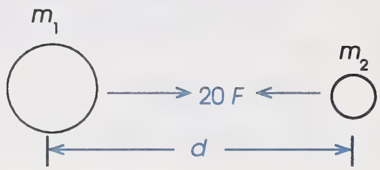
3. Write a proportion relating mass and force.

4. Newton showed that the force acts in the direction of a line connecting the centres of the masses. Illustrate this concept by drawing a diagram.

5. Consider the following diagram. A force of 10 F exists between Earth and a satellite. If the mass of the satellite increases three times, what force will now have to exist to keep the satellite in the same orbit?



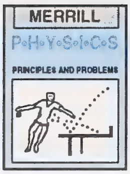
6. a. Consider the following diagram. The force between m_1 and m_2 is 20 F. When the distance d is increased to $4d$, how is F going to change?



b. When the distance d is changed to $\frac{1}{2}d$, what will F be?

7. Combine the two proportions relating F, d, and m.

The proportion that you wrote in question 7 describes the law of gravitation. On Earth, the gravitational force of attraction between two objects with masses of several kilograms each is very small. In 1798 a scientist named Henry Cavendish used a unique apparatus to measure the force that two lead spheres exert on each other.

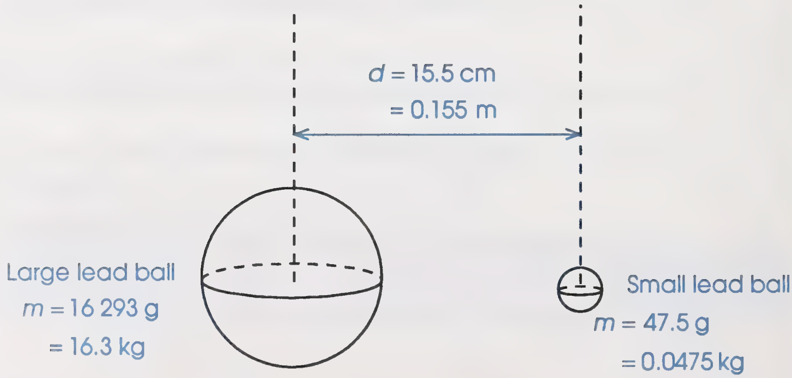


Refer to your textbook and read from the bottom of page 162 to the bottom of page 163 to find out more about the Cavendish experiment.

8. Why did Cavendish use lead spheres to measure the force of gravity?

9. Why did Cavendish keep the lead spheres enclosed?

Sample results from a Cavendish-type experiment are shown in the following diagram.



The force of gravitational attraction between the two lead balls was also measured.

$$F_g = 2.15 \times 10^{-9}\ \text{N}$$

If this data is substituted into Newton's law of universal gravitation, the only unknown quantity is the gravitational constant, G.

$$F_g = \frac{Gmm}{d^2}$$

$$2.15 \times 10^{-9}\ \text{N} = \frac{G(16.3\ \text{kg})(0.0475\ \text{kg})}{(0.155\ \text{m})^2}$$

$$2.15 \times 10^{-9}\ \text{N} = G(32.22\ \text{kg}^2/\text{m}^2)$$

$$G = \frac{2.15 \times 10^{-9}\ \text{N}}{(32.22\ \text{kg}^2/\text{m}^2)}$$

$$G = 6.67 \times 10^{-11}\ \frac{\text{N} \bullet \text{m}^2}{\text{kg}^2}$$

Notice that the gravitational constant does the following two important things for the law of universal gravitation.

- G allows both sides of the equation to be numerically equal.
- G allows the units on both sides of the equation to be equal.

The gravitational constant is a very small number. If you write it as a decimal number instead of using scientific notation, it looks like this:

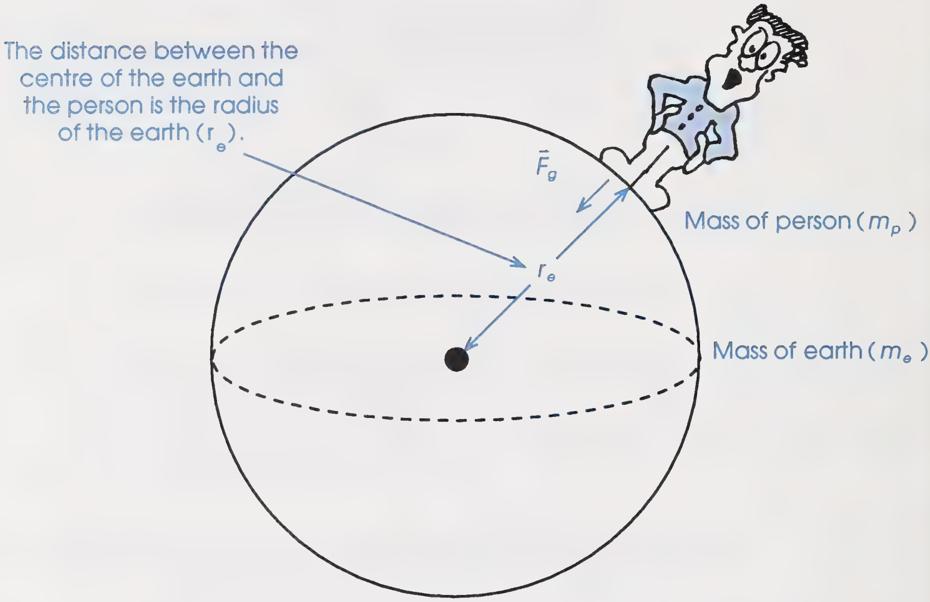
$$G = 6.67 \times 10^{-11}\ \frac{\text{N} \bullet \text{m}^2}{\text{kg}^2}$$

$$= 0.000\,000\,000\,066\,7\ \frac{\text{N} \bullet \text{m}^2}{\text{kg}^2}$$

10. Explain how this small value for G will influence the force of gravity that is calculated for two objects.

11. Calculate the force of gravity between two people that are standing 1.0 m apart. Assume that each person has a mass of 65 kg. Would either person be able to easily notice this force?

It is important not to confuse the gravitational constant (G) with the acceleration due to gravity (g). These two quantities are not the same, but they are related. Consider the two methods for calculating the force of gravity for a person standing on the surface of the earth.



Equation from Module 2

$$\boxed{F_g = m_p g}$$

Equation from Module 3

$$\boxed{F_g = \frac{Gm_e m_p}{(r_e)^2}}$$

Since the forces are the same in both calculations, the equations can be combined.

$$\boxed{m_p g = \frac{Gm_e m_p}{r_e^{\,2}}}$$

The mass of the person can be cancelled from both sides.

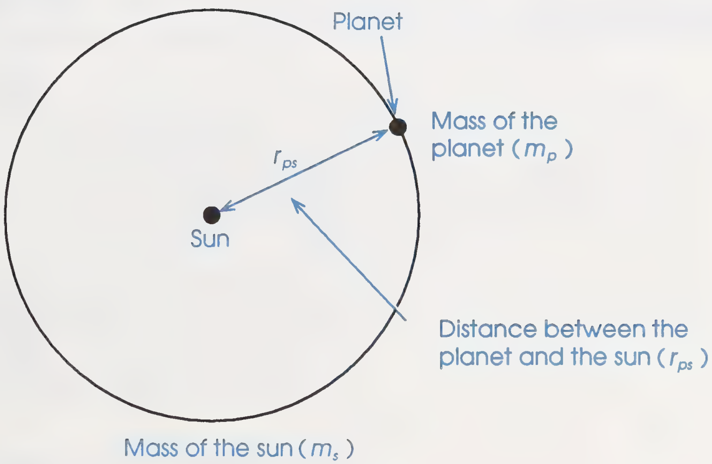
$$\boxed{g = \frac{Gm_e}{r_e^{\,2}}}$$

12. It was known at the time of Cavendish that the radius of Earth is 6.37×10^{6} m. Use this fact and the values for the acceleration due to gravity and the gravitational constant to calculate the mass of Earth.

13. Why is the experiment of Cavendish sometimes called the "weighing the earth" experiment?

14. Why is the acceleration due to gravity $9.8\ \text{m/s}^2$ for all objects on the surface of the earth, regardless of their mass?

15. Kepler's ideas on planetary motion can be re-interpreted using the gravitational constant. Carefully read the top half of page 162 and then complete the flow chart on the next page to solve for the period of a planet in orbit around the sun.



Assuming that the orbits can be treated as circles, a centripetal force is required.

The planet is held in its orbit by the force of gravity, as given by Newton's law of universal gravitation.

The force of gravity provides the centripetal force.

$$F_c = F_g$$

Substitution

Cancel the planet's mass.

Rearrange and solve for the period.

16. Refer to the data chart on page 159 in your textbook and to your answer for question 15 to calculate the period of the earth's orbit around the sun (in days).

17. Use your knowledge of circular motion and your answer for question 16 to calculate the speed of the earth as it orbits the sun.

Check your answers by turning to the Appendix, Section 3: Activity 3.

In the next activity you will discover how the law of universal gravitation can be applied to satellites that circle the earth.

Activity 4: Following a Satellite

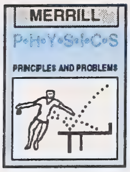
In this activity you will summarize key ideas from the whole module through a study of satellite motion. The photograph on the cover of this module booklet shows three astronauts on the space shuttle Endeavor capturing the Intelsat VI satellite as it orbits the earth. Satellites can be described and explained from the following points of view:

- satellites explained as projectiles
- satellites travel with circular motion
- satellites with humans on board
- satellites in gravitational fields

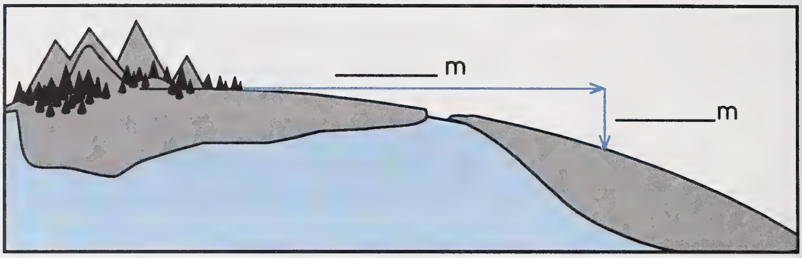
Satellites Explained as Projectiles

Isaac Newton made some interesting predictions about satellite motion almost 300 years before mankind launched the first artificial satellite. Newton's predictions were based on the technology of cannons and cannonballs since that was what was familiar to him.

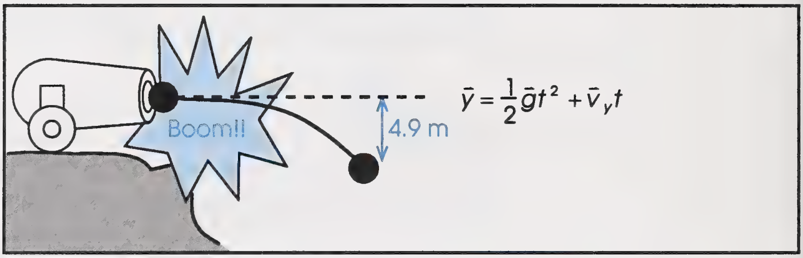
1. Carefully read page 164 and study Figure 8-9. Then summarize Newton's ideas on the diagrams on the next page. The diagrams are not to scale and they assume that air resistance can be ignored.



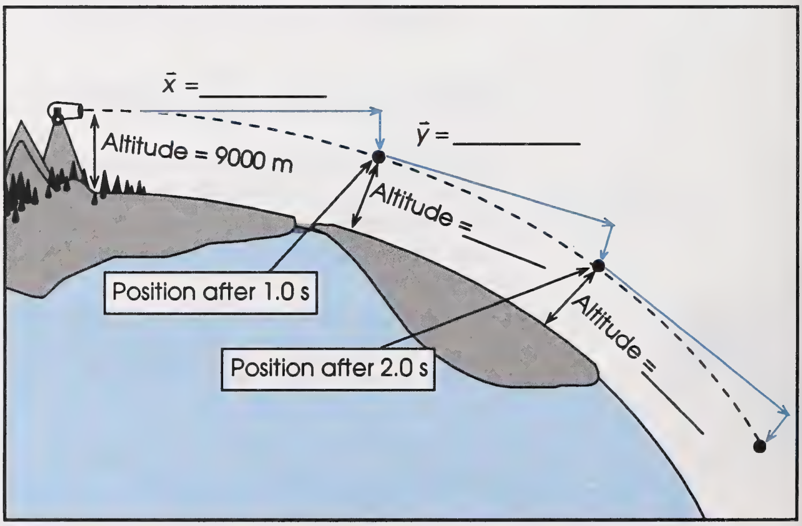
a. Label this diagram to show the curvature of the earth. Refer to Figure 8-9 in your textbook to help you answer.



b. No matter what the initial velocity is, a horizontally launched projectile will fall 4.9 m in 1 s. Complete the calculation that proves this fact.



c. Label this diagram to show the path of the cannonball projected horizontally at 8.0 km/s.

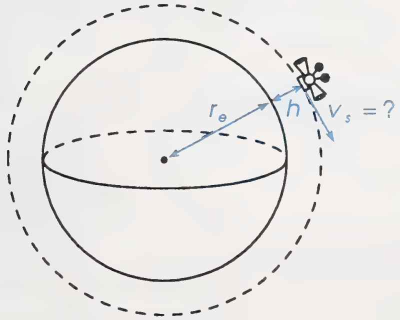


One way to summarize Newton's thoughts is to say that the cannonball keeps falling towards Earth, but Earth's surface is curving away at the same rate. The cannonball is left to fall at the same altitude forever!

In these circumstances the cannonball would become a satellite in orbit around the earth. Although modern satellites are not launched from cannons on mountain tops, the basic physics of projectile motion still applies. The key is for the satellite to maintain a horizontal velocity of about 8000 m/s.

Satellites Travel with Circular Motion

Another way to arrive at an estimate for the speed of a satellite in low orbit is to consider the situation from the pont of view of circular motion. The following description is for a satellite in orbit at an altitude of 225 km.



v_s (speed of satellite) = ?
r_e (radius of Earth) = 6.37×10^6 m
h (altitude of satellite) = 225 km = 2.25×10^5 m
m_e (mass of Earth) = 5.98×10^{24} kg
m_s (mass of satellite) = ?
r_s (radius of the satellite's orbit) $= r_e + h = 6.595 \times 10^6$ m

$$F_c = F_g$$

The force of gravity supplies the necessary centripetal force.

Substitution

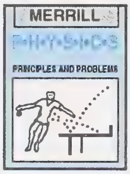
$$\frac{m_s v^2}{r_s} = \frac{G m_e m_s}{(r_s)^2}$$

Cancel the mass of the satellite and one radius.

$$v^2 = \frac{G m_e}{r_s}$$

Rearrange for speed.

$$v = \sqrt{\frac{G m_e}{r_s}}$$

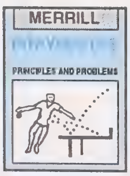


2. Solve for the speed of the satellite. Check your answer by reading the description, including the Example Problem, on page 165 of your textbook.

3. Would a satellite with more mass travel slower in its orbit around Earth? Explain your answer.

4. On the previous diagram of the satellite, draw in a vector to represent the force of gravity. Is your drawing consistent with what you know about centripetal force?

Check your answers by turning to the Appendix, Section 3: Activity 4.



5. Do Practice Problems 5, 6, and 8 on page 166 of your textbook. Be sure to start each problem with a statement of what supplies the centripetal force.

Check your answers by turning to pages 668 and 669 in your textbook.

Satellites with Humans on Board

When astronauts orbit the earth in the space shuttle or a space station, they are actually riding a satellite. To discover how this situation affects the astronauts, carefully read pages 154 and 155 of your textbook.

The word *weightlessness* is often misunderstood. Carefully read pages 166 and 167 of your textbook to discover the meaning of the word *weightlessness*.

6. Define *weightlessness*.

7. Weightlessness clearly has its advantages for an astronaut. Look at the photo on the cover of your module booklet. Can you think of some advantages of operating in a weightless environment?

Weightlessness also has its disadvantages for people, particularly when it comes to health. People will lose muscle tone, experience calcium deficiencies, and lose red blood cells after prolonged periods in weightless conditions.

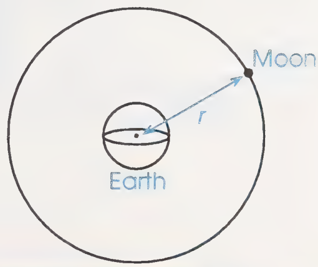
8. Suggest a reason why astronauts would lose muscle tone.

Satellites in Gravitational Fields

After the work of Newton and Cavendish, it was possible for people to perform calculations that could describe the motion of many observable objects in the heavens. For example, if your knew all the data, calculating the gravitational force that Earth exerts on the moon would not be that difficult. The moon is Earth's natural satellite.

9. Use the sketch and data provided to calculate the force of gravity on the moon.

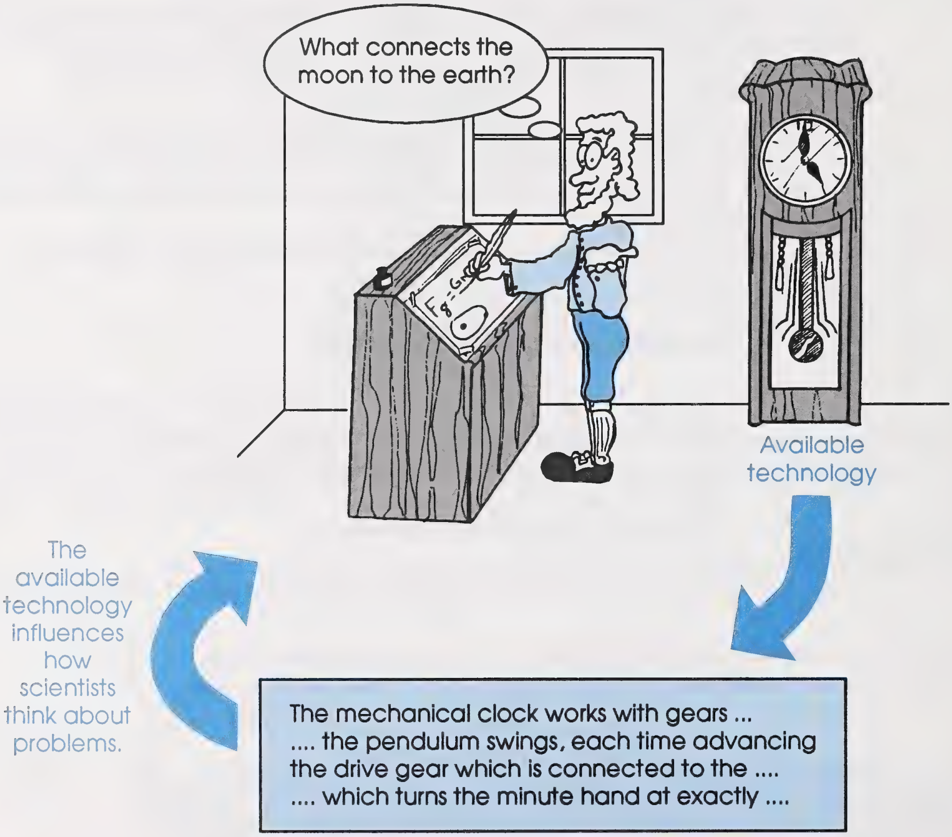
m_e (mass of the earth) $= 5.98 \times 10^{24}$ kg
m_m (mass of the moon) $= 7.34 \times 10^{22}$ kg
r (distance between Earth and moon)
$= 3.80 \times 10^{8}$ m
F_g (Earth's force of gravity on the moon) = ?



Check your answers by turning to the Appendix, Section 3: Activity 4.

Many people have difficulty explaining how gravitation happens. How could Earth exert such a huge force over the incredible distance between itself and its satellite, the moon?

Newton was uncomfortable with the idea that Earth could somehow exert a large force on the moon with no physical connection between them. For many years after Newton's death, other scientists shared this concern over the idea of action at a distance. These scientists lived in an age when all the advanced technologies were mechanical, and so they tended to look at problems from a mechanical point of view.

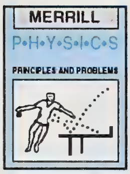


Since the scientists so strongly believed that there must be a connection between Earth and the moon, they invented such a connection.

The Connection Between Earth and the Moon	
Property	**Rationale**
The connection is invisible.	Nobody has ever seen this connection!
The connection is almost completely undetectable.	Unlike a large rod or wire holding the moon in place, this connection can't be touched and it has no mass.
The only way that this connection can be detected is to place a mass in it, and then it will exert a force on the mass.	When any object with a mass is placed above the surface of Earth and released, it is immediately forced to Earth by this connection.

gravitational field – the space surrounding a mass in which other masses within the region will experience a gravitational force

The name given to this invisible connection between Earth and the moon is the **gravitational field**.



Turn to page 168 of your textbook and examine the picture of Earth's gravitational field.

10. How does the picture show the strength of Earth's gravitational field at different locations?

The gravitational field of Earth extends out from Earth in all directions, allowing Earth to exert forces on everything from artificial satellites to the moon.

gravitational field at a point – represented by a vector that indicates the force per unit of mass

The gravitational field at a point in space can be represented with a vector, as given by the following equation.

$$\vec{g} = \frac{\vec{F}_g}{m}$$

The direction of the gravitational field is the same as the direction of the gravitational force on a test mass.

The magnitude of the gravitational field is determined by dividing the force of gravity on a test mass by the mass of the test object itself. The units are N/kg.

Gravitational field is just another name for the acceleration due to gravity. Earlier in this section you combined two equations to get a new description of the acceleration due to gravity on Earth. The same thinking can now be applied to the gravitational field of Earth.

$$F_g = mg$$

Rearrange.

$$g = \frac{F_g}{m}$$

$$F_g = \frac{Gm_e m}{(r_e)^2}$$

Substitution

$$g = \frac{\left(\frac{Gm_e m}{(r_e)^2}\right)}{m}$$

Cancel the mass of the object.

$$g = \frac{Gm_e}{(r_e)^2}$$

This equation makes some important points about what influences the magnitude of the gravitational field on the surface of Earth.

- The mass of Earth helps to determine how strong the gravitational field is.
- Since the gravitational constant is such a small number, only objects with large masses (planets, stars, and large moons) will produce a significant gravitational field.
- The further you are from Earth's centre, the smaller the value of the gravitational field will be.

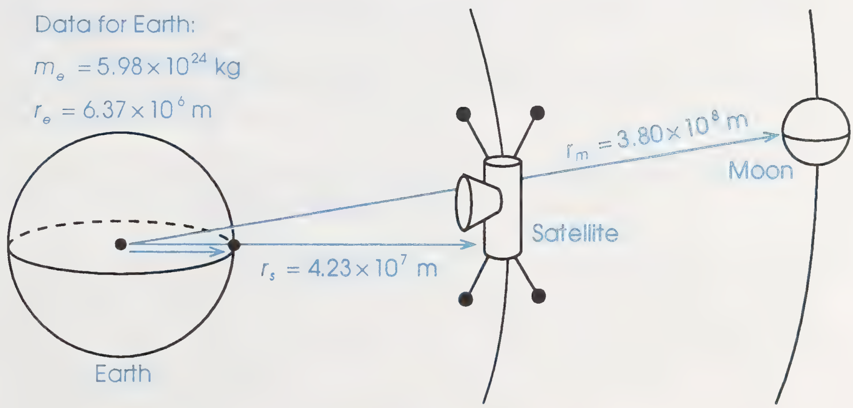
11. Use the data and information shown on the diagram to complete these calculations.

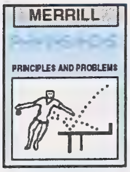
    a. Calculate the strength of the gravitational field at the surface of the earth.

    b. Calculate the strength of the gravitational field at the location of the **geosynchronous satellite**.

    c. Calculate the strength of the gravitational field at the location of the moon.

geosynchronous satellite – a satellite that remains above the same point on the earth's surface





12. Use the data in Table 8-1 on page 159 of your textbook to calculate the gravitational field strength on the surface of these planets.

    a. Mercury

    b. Mars

    c. Jupiter

13. Use your answers from questions 11 and 12 to calculate the force of gravity that would act on a 4.5-kg stone on the surface of these planets.

    a. Earth

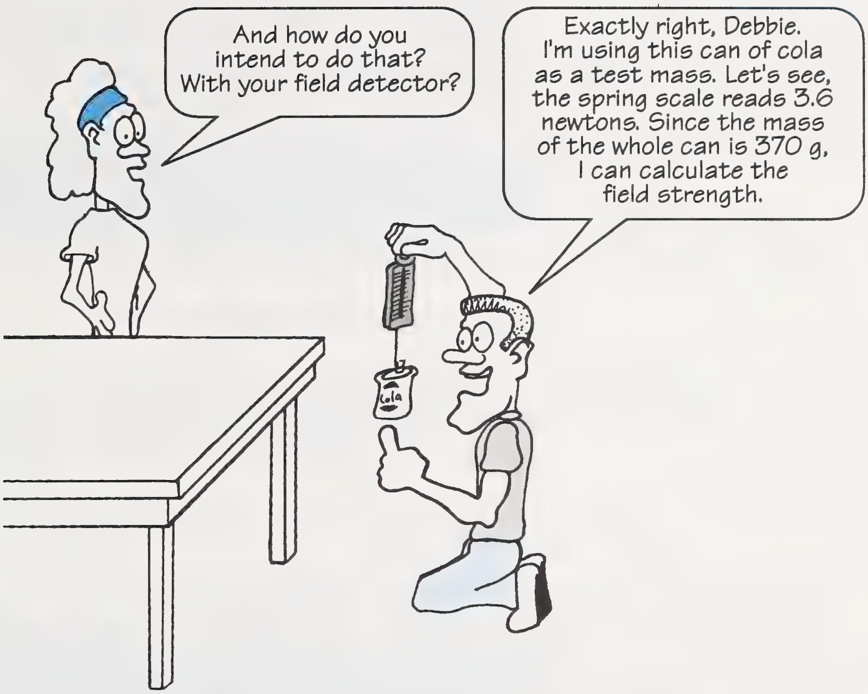
    b. Mercury

    c. Mars

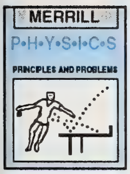
    d. Jupiter

14. Earlier in the course you learned that mass is a constant property of an object, while weight can vary from place to place. Use your answers from question 13 and the concept of gravitational field to reinterpret this idea.

Using the mass and radius of a planet is not the only way to calculate the gravitational field. Read the following brief account of how two students explored the gravitational field of Earth.



15. Complete Donald's calculation of the gravitational field strength using his measurements.



Carefully read from page 168 through to page 170 to discover more about gravitational fields and forces.

16. Einstein's insights into the nature of gravity are quite different from Newton's. Briefly outline Einstein's concept of gravity.

This activity began by thinking of satellites as projectiles falling to Earth. You were then introduced to satellites from a circular motion point of view, with the force of gravity providing the centripetal force. Finally, you learned that satellites could be seen as a consequence of a planet's gravitational field exerting a force on them.

17. Use Einstein's notion of gravity to explain satellite motion.

18. This is a good opportunity to summarize the main ideas in this module. Quickly skim through the pages of this module to identify the new equations that have been introduced. Record these equations under appropriate headings on the sheet that you've been using to summarize the equations from the whole course.

Check your answers by turning to the Appendix, Section 3: Activity 4.

Follow-up Activities

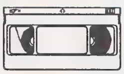
If you had difficulties understanding the concepts in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts, it is recommended that you do the Enrichment.

Extra Help

PATHWAYS

If you have access to the video called *The Apple and the Moon: The Mechanical Universe Series* (1985), do Part A. If you do not have access to the video, do Part B.

Part A



To review the main concepts in this module, watch the video called *The Apple and the Moon: The Mechanical Universe Series* (1985). Don't worry about the slightly different versions of the equations and the use of feet instead of metres in measurements. Focus on the historical development of the ideas and on the concepts related to the questions that follow. Be sure to read through the set of questions before you watch the video and then answer the questions when you finish watching the video.

1. Who was the scientist that said all objects accelerate downwards at the same rate?

2. When planets move around the sun, they follow elliptical orbits. Compare the speed of the planets when they are closest to the sun and furthest from the sun.

3. What do you call the point where the sun is positioned with respect to the elliptical orbit?

4. State Kepler's three laws as described on the video.

5. During the Apollo moon launch mission, what force had to be overcome at blast-off?

6. How does the force of gravity depend on the mass of two objects and their distance of separation? Express your answer as a proportion.

7. What does the value *G* represent?

8. The gravitational field on the moon is less than that on Earth. Compare the rates of fall for the hammer and the feather towards the moon's surface. Explain your answer.

9. What is the expression $\frac{Gm_e}{r_e^2}$ equal to?

10. When a mass is projected horizontally, the distance that it travels depends on what characteristic of the projectile's launch?

11. The astronaut in the spacecraft is trying to separate the water from the spoon. Explain why he is having difficulty doing this.

12. The distance to the moon is expressed as how many Earth radii?

13. On Earth, objects fall 4.9 m in 1 s. How far towards Earth does the moon fall in 1 s?

End of Part A

Part B

14. Complete the following summary chart for the equations and ideas used in this section.

Summary Chart for Kepler, Newton, and Gravitation		
Name of Equation or Idea	**Statement of Equation or Idea**	**Diagram to Illustrate the Meaning of the Variables**
Kepler's First Law		
Kepler's Second Law		
Kepler's Third Law		
Newton's Law of Universal Gravitation		
Gravitational Constant		

Summary Chart for Kepler, Newton and Gravitation		
Name of Equation or Idea	**Statement of Equation or Idea**	**Diagram to Illustrate the Meaning of the Variables**
Acceleration Due to Gravity		
Vertical Displacement of a Projectile		
Centripetal Force for Satellites		
Gravitational Field		

Check your answers by turning to the Appendix, Section 3: Extra Help.

Enrichment

Science Skills

- ☐ A. Initiating
- ☑ B. Collecting
- ☑ C. Organizing
- ☐ D. Analysing
- ☐ E. Synthesizing
- ☐ F. Evaluating

Do **one** of the following activities.

1. Library Research

   In the first activity of this section you learned about the Moose Mountain Medicine Wheel and the possibility of its use as an astronomical observatory. Other examples of astronomy done by native people are the cities and temples built by the Mayan civilization in Central America. The Mayans built observatories and developed a system of arithmetic that surpassed anything similar that was happening at the same time in Europe.

   Use the resources at your local library to find answers to the following questions.

   a. When did the Mayan cities flourish?

   b. The most famous Mayan observatory was the Caracol tower at Chichén Itzá in Yucatán. What two objects in the sky was this tower designed to observe?

   c. The Mayan calendar was based on a 584-day year and a 365-day year. What is the significance of these two numbers in terms of astronomical observations?

   d. What was the practical value of these observations to this civilization?

2. A Constant for Planetary Motion

   In Activity 3 of this section you re-interpreted Kepler's laws in terms of the law of universal gravitation. Another approach is to derive an equation that equates $\frac{r^3}{T^2}$ to a constant.

   a. Beginning with the idea that the force of gravity supplies the centripetal force, derive an equation that shows that $\frac{r^3}{T^2}$ is a constant.

   b. Determine the value of the constant.

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PRINCIPLES AND PROBLEMS

c. Refer to Table 8-1 on page 159 of your textbook to determine the value of $\frac{r^3}{T^2}$ for Earth, Venus, and Mars. The period for Venus is 225 days and the period for Mars is 687 days.

d. Compare your answers to parts b. and c. Are they consistent?

Check your answers by turning to the Appendix, Section 3: Enrichment.

Conclusion

You now have an understanding of Kepler's laws and Newton's law of universal gravitation, both of which describe orbiting objects. This is particularly helpful when applied to the motion of satellites, which can be thought of as projectiles, objects moving in circles, or masses held in orbit by Earth's gravitational field.

Einstein's thoughts on the nature of gravity are particularly helpful as a reminder that many of the presently accepted theories about gravity are merely stepping stones towards a deeper and more profound understanding.

Assignment Booklet

ASSIGNMENT

Turn to your Assignment Booklet and do the assignment for Section 3.

MODULE SUMMARY

You should now realize that many objects in daily life can be considered to be projectiles or objects which undergo uniform circular motion. The motion of these objects can be analysed using Newton's laws and the concepts of uniform and accelerated motion. These objects are influenced by the effects of gravity, a force which affects all masses.

In the next module you will examine the energies possessed by resting and moving objects. You will learn to re-interpret uniform circular motion as an example of simple harmonic motion.

Appendix

Glossary

Activities

Extra Help

Enrichment

Glossary

astronomer – a scientist who studies astronomy

astronomy – the study of the brightness, motion and composition of objects in the heavens

celestial bodies – bodies in the sky or visible heavens

centrifugal force – a fictitious force used by an observer in a rotating reference frame to explain observations

centripetal acceleration – the acceleration directed toward the center of a circle

centripetal force – the force that is directed towards the centre of circular motion

dimensional analysis – the process of using units to help develop an equation

ellipse – a closed plane curve formed by the elongation of a circle

focus – one of two fixed points used in the creation of an ellipse

geosynchronous satellite – a satellite that remains above the same point on the earth's surface

gravitational constant – the physical value needed in Newton's law of gravitation to determine the actual force between two bodies

gravitational field – the space surrounding a mass in which other masses within the region will experience a gravitational force

gravitational field at a point – represented by a vector that indicates the force per unit of mass

gravitational force – a force of attraction between two bodies due to their masses

height – the maximum vertical displacement of a projectile

lab frame of reference – The observer is attached to the ground, or the lab, and is not attached to what is observed to be moving.

Newton's law of universal gravitation – the law which describes the attraction between two masses

parabola – a curve generated on a plane by a point moving such that its distance from a fixed point is equal to its distance from a fixed line

period – the time taken for one complete rotation

projectile – an object launched through the air or some other medium

range – the maximum horizontal displacement of a projectile

rotating frame of reference – The observer is attached to the rotating object and makes observations from this point of view.

trajectory – the path followed by a projectile

weightlessness – a condition in which the object being weighed is in free fall, so the scale being used reports a zero force

Suggested Answers

Section 1: Activity 1

1. Both balls struck the floor at the same time.

2. The horizontal velocity of the ball does not determine how long it takes to hit the floor. The important factor seems to be the acceleration due to gravity. Since g equals $9.80\ \text{m/s}^2$ for both balls, they strike the ground at the same time.

3. The sound indicates that the pennies hit the floor at the same time.

4. No, there is no difference. The pennies still strike the floor at the same time.

5. No, the speed of the flicked penny does not determine when it hits the floor because it always takes the same amount of time as the nudged penny.

6. It would seem that the acceleration due to gravity determines how long the penny takes to strike the floor. Since g is $9.80\ \text{m/s}^2$ for both pennies, they strike the floor at the same time.

7. Only when air resistance is considered to be negligible can the horizontal motion be uniform. According to Newton's second law, if air resistance was significant, there would be a force slowing the object down. Only when air resistance is considered to be negligible will the vertical motion be uniformly accelerated at $9.80\ \text{m/s}^2$.

8. This situation is identical to the balls on the laser videodisc and the pennies on the table. Even if the speed of the bullet was 350 m/s, gravity still acts on it. Since g is $9.80\ \text{m/s}^2$ for both bullets, they strike the ground at the same time.

Section 1: Activity 2

1. Answers are shown on the data chart following question 3.

2. Answers are shown on the data chart following question 3.

3. Answers are shown on the following data chart. Sample calculations are shown.

Average horizontal speed:

$$v_x = \frac{x}{t} = \frac{2.0\text{ cm}}{1\text{ tock}}$$

$$v_x = 2.0\text{ cm/tock}$$

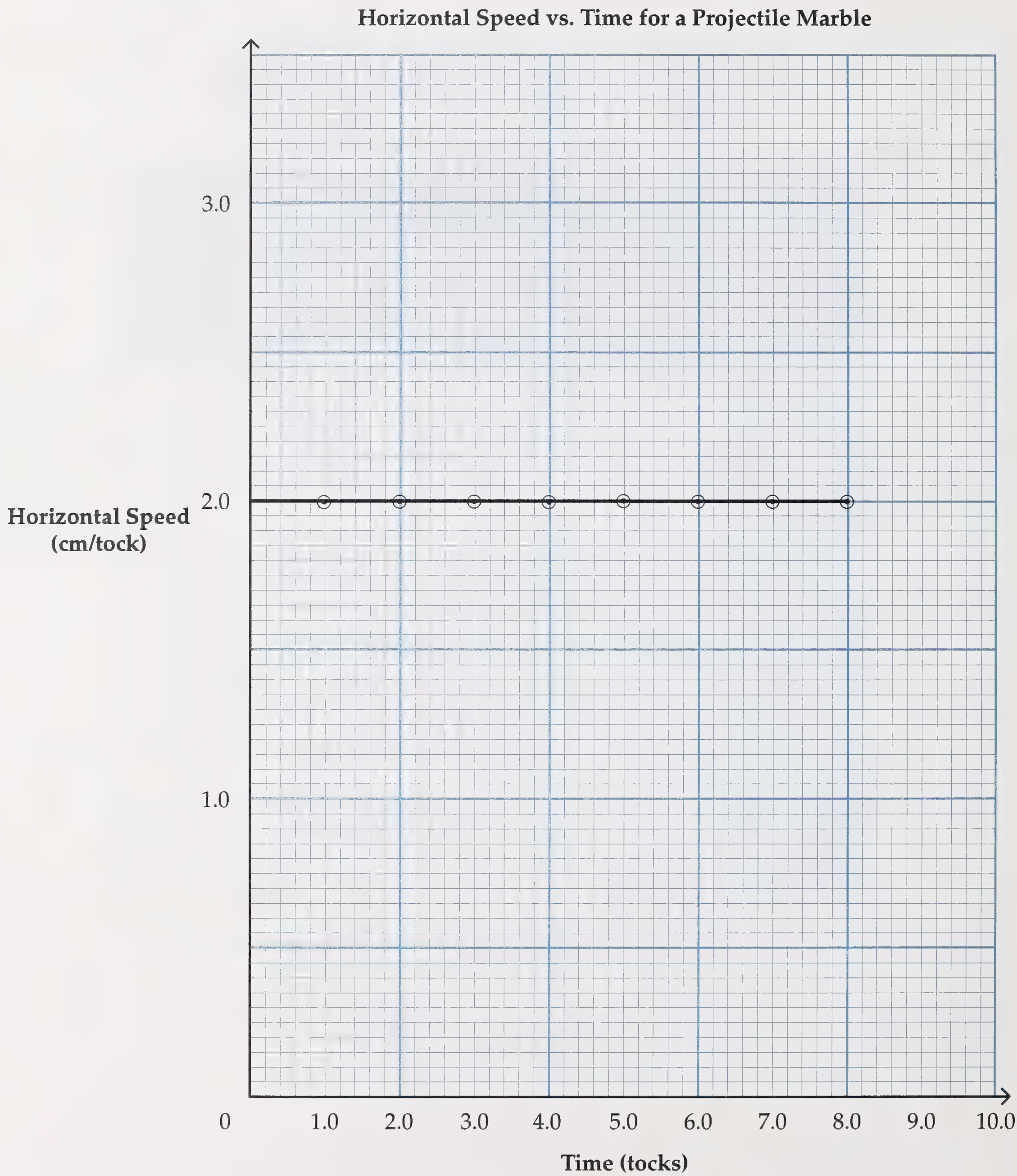
Average vertical speed:

$$v_{yf} = \frac{2y}{t} = \frac{2(1.8\text{ cm})}{4.0\text{ tocks}}$$

$$v_{yf} = 0.90\text{ cm/tock}$$

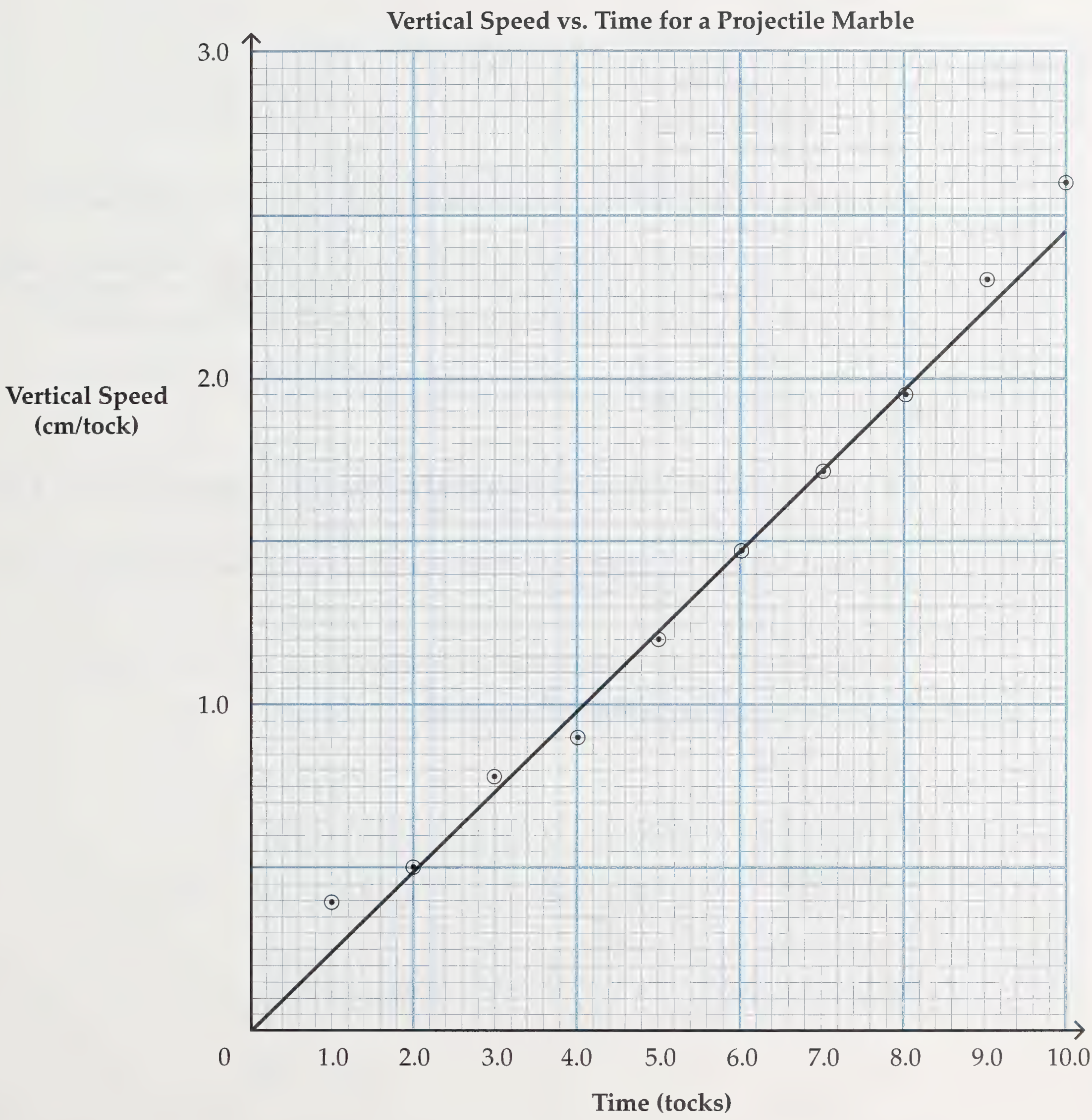
Data for Projectile Marble				
Time Interval (tock)	**Horizontal Distance *x* (cm)**	**Average Horizontal Speed (cm/tock)**	**Vertical Distance *y* (cm)**	**Average Vertical Speed (cm/tock)**
0	0	0	0	0
1	2.0	2.0	0.2	0.40
2	4.0	2.0	0.5	0.50
3	6.0	2.0	1.0	0.77
4	8.0	2.0	1.8	0.90
5	10.0	2.0	3.0	1.20
6	12.0	2.0	4.4	1.47
7	14.0	2.0	6.0	1.71
8	16.0	2.0	7.8	1.95
9	18.0	2.0	10.3	2.29
10	20.0	2.0	13.0	2.60

4. This graph is based on the sample data. **Answers will vary, but in all cases the graph should be flat.**



5. The horizontal speed-time graph shows uniform motion. While the marble is travelling horizontally across the inclined surface, it is travelling at a constant speed of 2.0 cm/tock.

6. This graph is based on the sample data. Answers will vary, but the result should be a sloped straight line.



7. The vertical speed of the marble is increasing as it moves down the incline because the vertical speed-time graph rises to the right. The horizontal speed is constant.

8. Calculation of the slope of the vertical speed-time graph:

$$\begin{aligned}\text{slope} &= \frac{\text{rise}}{\text{run}} \\ &= \frac{2.44 \text{ cm/tock}}{10.0 \text{ tock}} \\ &= 0.24 \text{ cm/tock}^2\end{aligned}$$

The value of the slope of the vertical speed-time graph represents acceleration.

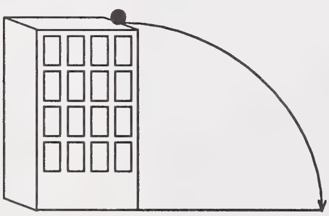
9. When the marble rolls down the sloped surface, it has a constant horizontal velocity. The vertical velocity increases in the downward direction. These two motions are totally independent of each other and produce a path that is shaped like a parabola.

Section 1: Activity 3

1. The equation used to describe the horizontal motion of a projectile is $\vec{d} = \vec{v}t$ or $\vec{x} = \vec{v}_x t$.

2. The equation used to describe the vertical motion of a projectile is $\vec{d} = \vec{v}_i t + \frac{1}{2}\vec{a}t^2$ or $\vec{y} = \vec{v}_y t + \frac{1}{2}\vec{g}t^2$.

3. The sketch illustrates the path of the baseball.



4. Horizontal motion:

- $\vec{d} = \vec{v}t$
- $\vec{x} = \vec{v}_x t$

Vertical motion:

- $\vec{d} = \vec{v}_i t + \frac{1}{2}\vec{a}t^2$
- $\vec{y} = \vec{v}_y t + \frac{1}{2}\vec{g}t^2$

The variable t (time) is the same in both equations.

5. The manipulation of the equation $\vec{y} = \vec{v}_y t + \frac{1}{2}\vec{g}t^2$ to solve for the variable t is shown.

$$\vec{y} = \vec{v}_y t + \frac{1}{2}\vec{g}t^2 \qquad \vec{v}_y = 0\text{, therefore } \vec{v}_y t = 0.$$

$$\vec{y} = \frac{1}{2}\vec{g}t^2$$

$$t^2 = \frac{2\vec{y}}{\vec{g}}$$

$$t = \sqrt{\frac{2\vec{y}}{\vec{g}}}$$

6. The substitution of known values into the equation is shown.

$$t = \sqrt{\frac{2(-20\text{ m})}{-9.80\text{ m/s}^2}}$$

7. Now it is possible to solve for t.

$$t = \sqrt{\frac{-40\text{ m}}{-9.80\text{ m/s}^2}}$$

$$t = \sqrt{4.08\text{ s}}$$

$$t = 2.0\text{ s}$$

8. The substitution and solution for the horizontal displacement x is shown.

$$\vec{x} = \vec{v}_x t$$

$$\vec{x} = 8.0\text{ m/s}(2.0\text{ s})$$

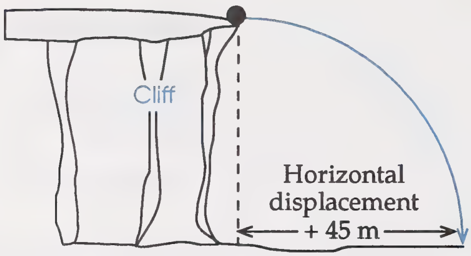
$$\vec{x} = 16\text{ m}$$

The baseball lands 16 m from the base of the building.

9. a. The initial velocity of the stone is zero $\left(\vec{v}_y = 0\right)$.

   b. The equation used to determine the time in flight is $\vec{y} = \vec{v}_y t + \frac{1}{2}\vec{g}t^2$ or $t = \sqrt{\frac{2\vec{y}}{\vec{g}}}$.

   c.



10. Answers to these questions are found on page 667 of the textbook.

Section 1: Activity 4

1. a.

   b. Each half of the trajectory is similar to the path of the marble.

2. The horizontal component of the velocity is considered to be constant if air resistance is ignored.

3. The vertical component of the velocity is reduced until it is zero at the top or midpoint of the path. From the top of the path, the velocity increases in the negative direction as it approaches the ground. For the whole flight, the changes in the vertical component of the velocity are given by $g = -9.80\ \text{m/s}^2$.

4. a. The time taken for the projectile to reach its maximum height is equal to the time taken to fall back down to the same height as the launch position because the acceleration due to gravity is the same for the projectile when it rises and when it falls.

   b. When air resistance is ignored, the horizontal distances travelled during each half of the trajectory are equal.

5. Note that throughout these calculations an extra significant digit was used for preliminary calculations.

   a.
$$\begin{aligned} v_x &= \cos\theta(v_i) \\ &= (\cos 40°)(16\ \text{m/s}) \\ &= 12.3\ \text{m/s} \\ &= 12\ \text{m/s} \end{aligned}$$
$$\begin{aligned} v_y &= \sin\theta(v_i) \\ &= (\sin 40°)(16\ \text{m/s}) \\ &= 10.3\ \text{m/s} \\ &= 10\ \text{m/s} \end{aligned}$$

   b.
$$\begin{aligned} t &= \frac{-2\vec{v}_y}{\vec{g}} \\ &= \frac{-2(10.3\ \text{m/s})}{-9.80\ \text{m/s}^2} \\ &= 2.10\ \text{s} \\ &= 2.1\ \text{s} \end{aligned}$$

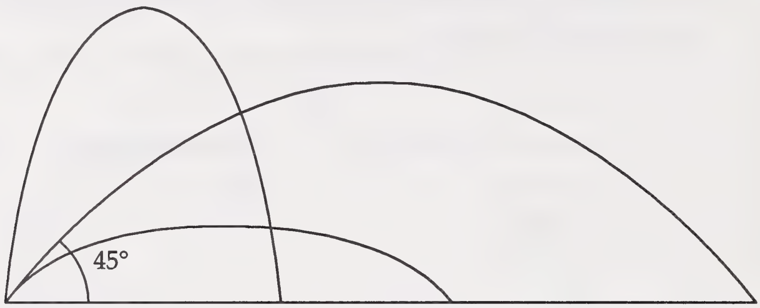
   c.
$$t = 2.10\ \text{s} \div 2 = 1.05\ \text{s}$$
$$\begin{aligned} \vec{y} &= \vec{v}_y t + \frac{1}{2}\vec{g}t^2 \\ &= (10.3\ \text{m/s})(1.05\ \text{s}) + \frac{1}{2}(-9.80\ \text{m/s}^2)(1.05\ \text{s})^2 \\ &= 10.815\ \text{m} + (-5.40225\ \text{m}) \\ &= 5.41\ \text{m} \\ &= 5.4\ \text{m} \end{aligned}$$

d. $t = 2.10\text{ s}$

$$\vec{x} = \vec{v}_x t$$
$$= (12.3\text{ m/s})(2.10\text{ s})$$
$$= 25.8\text{ m}$$
$$= 26\text{ m}$$

6. Answers to these problems can be found on page 667 in textbook.

7. Predictions about the angle of projection may vary, but the greatest range would occur at an angle of 45° because the horizontal and vertical velocities are equal at this angle.

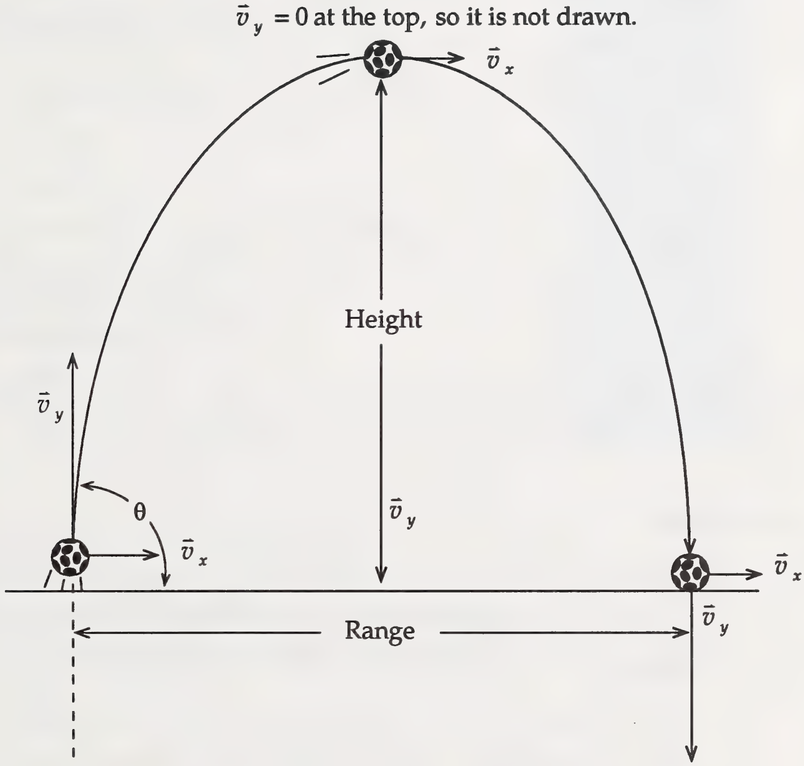


Section 1: Follow-up Activities

Extra Help

1. a. Answers are shown on the sketch following 1. b.

   b. Answers are shown on the following chart.



   c. All the horizontal velocity vectors should be the same length.

   d. The vertical velocity vectors should be getting shorter on the way up and longer on the way down.

2. The horizontal motion is uniform. The vertical motion is accelerated due to the effects of gravity. Both motions can be analysed independently of each other.

3.

Summary Chart for Projectile Motion		
	Horizontal Components	**Vertical Components**
Type of Motion	Uniform motion	Accelerated motion
Equation for Velocity Components	$v_x = \cos\theta(v_i)$ $\vec{v}_x = \frac{\vec{x}}{t}$	$v_y = \sin\theta(v_y)$
Other Useful Equations	Range: $\vec{x} = \vec{v}_x t$	$\vec{g} = \frac{\Delta\vec{v}_y}{t}$ Height: $\vec{y} = \vec{v}_y t + \frac{1}{2}\vec{g}t^2$ Time for whole flight when take-off and landing are at the same height: $t = \frac{-2\vec{v}_y}{\vec{g}}$

Enrichment

1. a. It is advantageous for frogs and kangaroos to jump at 45° from the ground because this gives them maximum range. This would allow them to travel the greatest distance with the least amount of energy expenditure and allow them to escape predators.

   b. Organisms that jump have large muscular thighs and relatively large feet. Large feet distribute the necessary "jumping force" over a large surface area. Jumping animals are usually herbivores. Their means of movement allows them to move easily between food sources and escape from predators.

2. a. Sample data and calculation of the speed of the thrown ball:

Distance (m)	Time (s)
12.2	2.6
23.5	3.0

Calculation of v_i (speed of the ball):

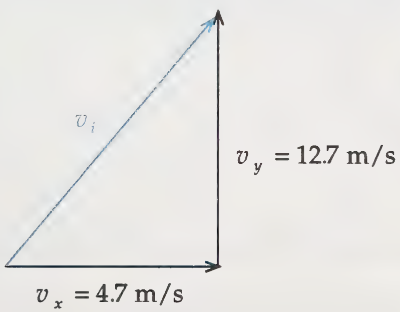
- Determine v_x.

$$x = v_x t$$
$$v_x = \frac{x}{t}$$
$$v_x = \frac{12.2\text{ m}}{2.6\text{ s}}$$
$$v_x = 4.7\text{ m/s}$$

- Determine v_y by first realizing that the ball is at its maximum height at half the total time in flight.

$$v_y = gt$$
$$v_y = \left(9.80\text{ m/s}^2\right)\left(1.3\text{ s}\right)$$
$$v_y = 12.7\text{ m/s}$$

- Sketch a right angle triangle and determine v_i using the Pythagorean theorem.



$$v_i^2 = v_x^2 + v_y^2$$
$$v_i^2 = 22.1\text{ m}^2/\text{s}^2 + 161.3\text{ m}^2/\text{s}^2$$
$$v_i^2 = 183.4\text{ m}^2/\text{s}^2$$
$$v_i = 13.5\text{ m/s}$$
$$v_i = 14\text{ m/s}$$

b. Textbook question 1:

Different students will throw the ball different ranges.

Textbook question 2:

Different students will throw the ball at different speeds.

c. Textbook question 1:

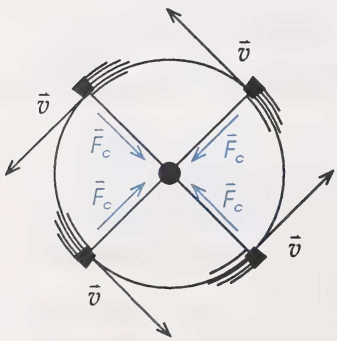
The thrower should try to throw the ball at 45° above the ground. This will result in the horizontal and the vertical velocities being equal and, as a result, the ball will travel the greatest distance.

d. Textbook question 1:

During the kick-off of a football game, it is important that the ball travels the greatest distance. During a punt in football, it is important that the ball remains in the air for the longest possible time so that the players can move downfield in the direction in which the ball is pointed.

Section 2: Activity 1

1. The string attached to the stopper keeps the stopper moving in a circle. If you were to suddenly release a length of string or cut the string, the stopper would fly off on a tangent.

2. 

3. The velocity and the centripetal force are at right angles to each other.

4. Higher speeds require more force. Lower speeds require less force.

5. To produce a smaller radius, more force is required. To produce a larger radius, a reduction in force is required.

6. Putting a heavier mass on the end of the string will require more force to keep it moving in a circle if speed and radius are kept constant. One example of this is swinging a small child in a circle by the arms. Children who have more mass require more force to move them in the same circle with the same speed.

7. Squaring the unit for speed creates the desired effect.

$$\frac{\mathrm{kg}(\mathrm{m/s})^2}{\mathrm{m}} = \frac{\mathrm{kg}\left(\mathrm{m}^2/\mathrm{s}^2\right)}{\mathrm{m}} = \frac{\mathrm{kg}\left(\mathrm{m}^{\cancel{2}}/\mathrm{s}^2\right)}{\cancel{\mathrm{m}}} = \frac{\mathrm{kg}(\mathrm{m})}{\mathrm{s}^2}$$

8. Since squaring the unit for speed produced the proper unit, squaring the speed could also produce the right equation.

$$F_c = \frac{mv^2}{r}$$

Section 2: Activity 2

1.
2. The following data was collected using a 10-g rubber stopper and washers that had a mass of 8 g each.
3.

Measurements and Calculations for the Circular Motion of a Stopper							
Trial	**Number of Washers**	**Time for 30 Revolutions (s)**	**Time for One Revolution (s)**	**Speed (m/s)**	**Mass of the Stopper (kg)**	**$\frac{mv^2}{r}$ (N)**	**F_c Supplied by Washers (N)**
1	4	23.7	0.79	3.97	0.010	0.32	0.31
2	8	16.2	0.54	5.81	0.010	0.68	0.63
3	12	13.8	0.46	6.83	0.010	0.93	0.94
4	16	12.0	0.40	7.85	0.010	1.2	1.3
5	20	10.5	0.35	8.97	0.010	1.6	1.6

4. These calculations are based on the sample data provided. Answers may vary.

Percent Error for Each Trial			
Trial	Experimental Value (N)	Theoretical Value (N)	Percent Error = $\frac{\lvert \text{Experimental} - \text{Theoretical} \rvert}{\text{Theoretical}} \times 100\%$
1	0.32	0.31	$\frac{\lvert 0.32 - 0.31 \rvert}{0.32} \times 100\% = 3.1\%$
2	0.68	0.63	$\frac{\lvert 0.68 - 0.63 \rvert}{0.68} \times 100\% = 7.4\%$
3	0.93	0.94	$\frac{\lvert 0.93 - 0.94 \rvert}{0.93} \times 100\% = 1.1\%$
4	1.2	1.3	$\frac{\lvert 1.2 - 1.3 \rvert}{1.2} \times 100\% = 8.3\%$
5	1.6	1.6	0%

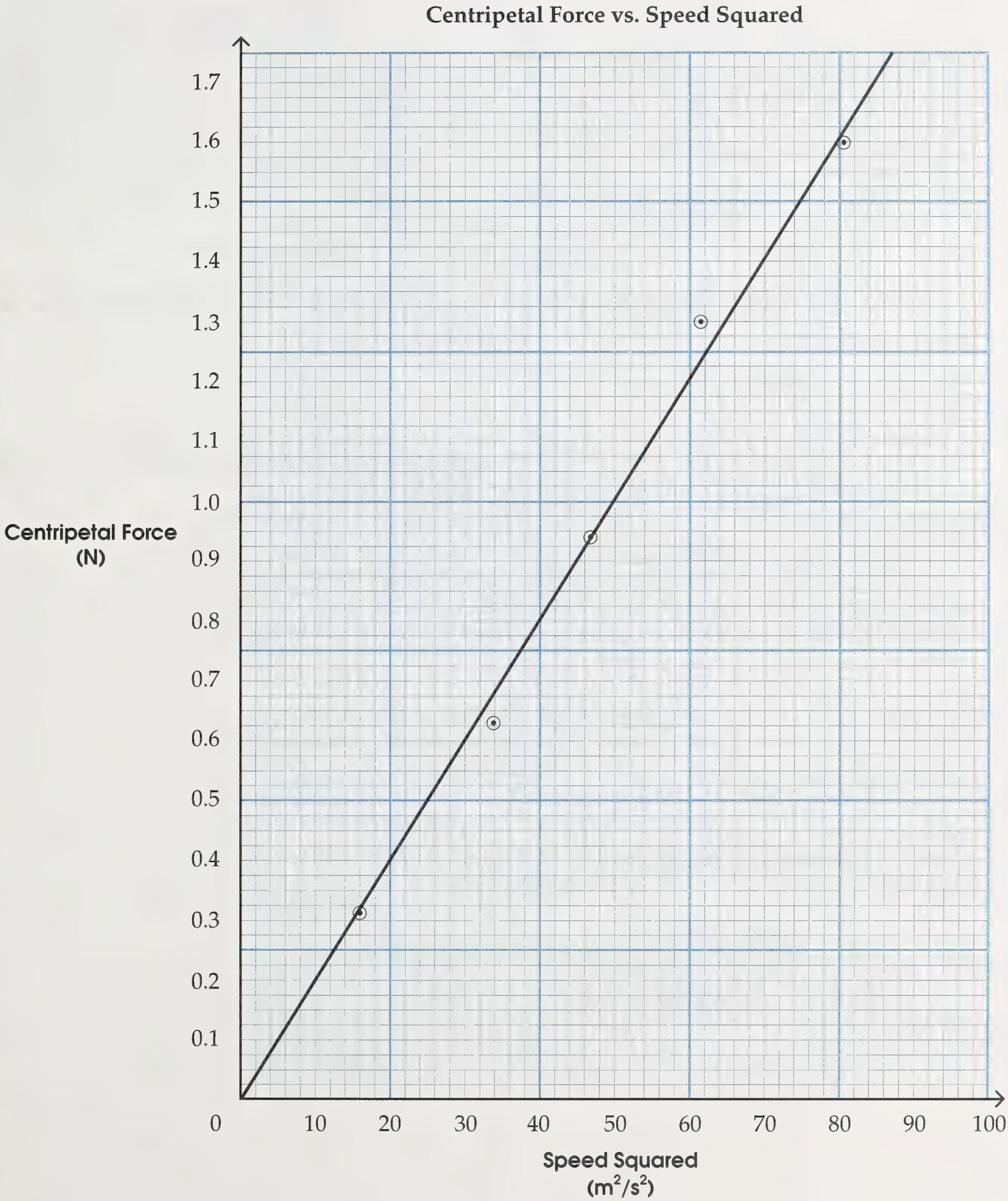
5. $$\text{average percent error} = \frac{\text{sum of all}}{\text{total number}} = \frac{19.9\%}{5} = 4.0\%$$

6. This percent error would allow you to say that $\frac{mv^2}{r}$ does equal F_c. Your answers may vary from this and your percent error may reach as high as 15 percent or 20 percent.

7.

Data for Graph			
Trial	v (m/s)	v^2 (m^2/s^2)	F_c (Supplied by Washers) (N)
1	3.97	15.8	0.31
2	5.81	33.8	0.63
3	6.83	46.6	0.94
4	7.85	61.6	1.3
5	8.97	80.5	1.6

8. This graph is based on the sample data provided. Answers may vary.



9. The following calculation is based on the sample data.

$$\begin{aligned}\text{slope} &= \frac{\text{rise}}{\text{run}} \\ &= \frac{1.725\ \text{N} - 0\ \text{N}}{86\ \text{m}^2/\text{s}^2 - 0\ \text{m}^2/\text{s}^2} \\ &= 0.020\ 058\ \frac{\text{N}}{\text{m}^2/\text{s}^2} \\ &= 0.020\ \frac{\text{N}}{\text{m}^2/\text{s}^2}\end{aligned}$$

10. The slope of this graph represents the ratio of centripetal force to the speed squared.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{F_c}{v^2}$$

If you substitute the appropriate quantities into this equation, you will find that the slope represents the ratio of mass to radius.

$$\begin{aligned}\text{slope} = \frac{\text{rise}}{\text{run}} &= \frac{F_c}{v^2} \\ &= \frac{\frac{mv^2}{r}}{v^2} \\ &= \left(\frac{mv^2}{r}\right)\left(\frac{1}{v^2}\right) \\ &= \frac{m}{r}\end{aligned}$$

11. Theoretical values:

$$\begin{aligned}\frac{\text{mass}}{\text{radius}} &= \frac{0.010\ \text{kg}}{0.50\ \text{m}} \\ &= 0.020\ \text{kg/m}\end{aligned}$$

Experimental values:

$$
\begin{aligned}
\frac{\text{mass}}{\text{radius}} &= \frac{0.020\ \text{N}}{\text{m}^2/\text{s}^2} \\
&= \frac{0.020\left(\text{kg} \bullet \text{m/s}^2\right)}{\text{m}^2/\text{s}^2} \\
&= 0.020\ \text{kg/m}
\end{aligned}
$$

$$
\begin{aligned}
\text{percent error} &= \frac{|\text{theoretical value} - \text{experimental value}|}{\text{theoretical value}} \times 100\% \\
&= \frac{0.020\ \text{kg/m} - 0.020\ \text{kg/m}}{0.020\ \text{kg/m}} \times 100\% \\
&= 0\%
\end{aligned}
$$

12. In this case the percent error is ideal. Acceptable percent errors for this method would be ten percent to 15 percent or less.

13. The graphical method takes longer and requires some interpretation of the slope, which may be complicated. However, the advantages of this method are numerous. The graphical method works in all circumstances. The process of drawing the line is actually an averaging technique itself, which tends to reduce the calculated percent error. The graphing technique also allows you to compensate for data points that are off the line.

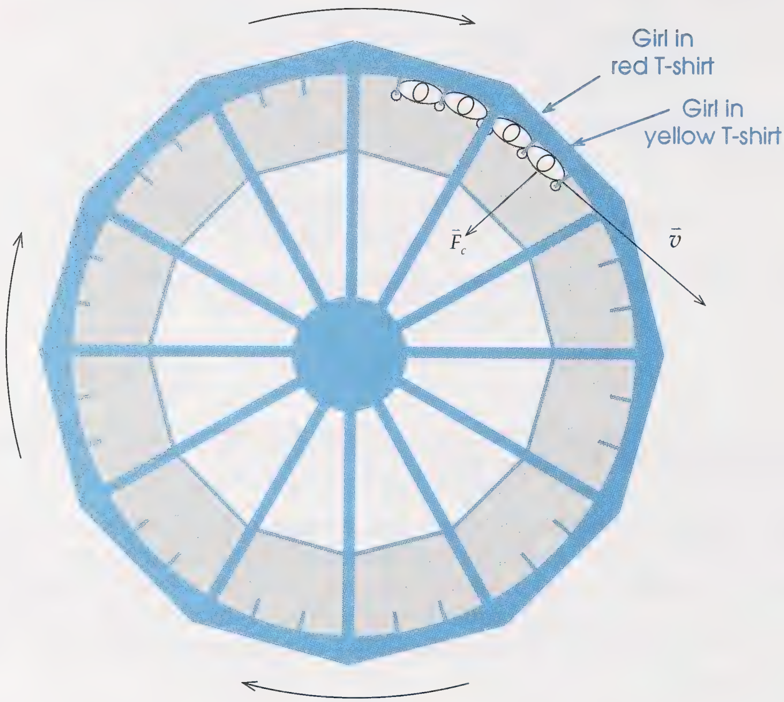
14. The data averaging method considers all the data points, but when the total is found, the larger values have a greater effect on the average then the smaller values. This means that an error will have a bigger impact on the result if it occurs on a larger value. The graphical method is superior in this respect because when the best fit line is drawn, all the data points are treated equally. Another advantage is that a point far from the trend can be disregarded in the graphical method. The graphical method will likely produce a different result than the data averaging method, but, in general, the graphical results are usually better.

15. Answers will vary for this question, as it depends on the data collected. In most cases the data should confirm the equation. Errors are likely due to the technique in swinging the rubber stopper, as it is difficult to keep the swing constant. Other errors could occur in measuring the radius, timing 30 revolutions, or determining the mass of the stopper and washers.

Section 2: Activity 3

1. The answer is shown on the diagram following question 2.

2.



3. $m = 52.5\ \text{kg}$

   $r = 4.1\ \text{m}$

   $$T = \frac{6.2\ \text{s}}{2}$$
   $$= 3.1\ \text{s}$$

   $F_c = ?$

   Step 1:

   $$v = \frac{2\pi r}{T}$$
   $$= \frac{2\pi(4.1\ \text{m})}{(3.1\ \text{s})}$$
   $$= 8.31\ \text{m/s}$$

   Note that speed was not rounded off here.

Step 2:

$$
\begin{aligned}
F_c &= \frac{mv^2}{r} \\
&= \frac{(52.5\ \text{kg})(8.31\ \text{m/s})^2}{4.1\ \text{m}} \\
&= 884\ \text{N} \\
&= 8.8 \times 10^2\ \text{N}
\end{aligned}
$$

4. $m = 52.5\ \text{kg}$

$r = 4.1\ \text{m}$

$$
\begin{aligned}
T &= \frac{6.2\ \text{s}}{2} \\
&= 3.1\ \text{s}
\end{aligned}
$$

$F_c = ?$

$$
\begin{aligned}
F_c &= \frac{m4\pi^2 r}{T^2} \\
&= \frac{(52.5\ \text{kg})(4)(\pi^2)(4.1\ \text{m})}{(3.1\ \text{s})^2} \\
&= 884\ \text{N} \\
&= 8.8 \times 10^2\ \text{N}
\end{aligned}
$$

5. The cage pushes on the person. The cage is held in place by the steel pipes and supports that hold the entire ride together.

6. Method 1 – Using $a_c = \dfrac{v^2}{r}$:

$$
\begin{aligned}
a_c &= \frac{v^2}{r} \\
&= \frac{\left(\frac{2\pi r}{T}\right)^2}{r} \\
&= \frac{4\pi^2 r^2}{rT^2} \\
a_c &= \frac{4\pi^2 r}{T^2}
\end{aligned}
$$

Method 2 – Using Newton's second law:

According to Newton's second law, all net forces cause masses to accelerate in the direction of the force. It follows that the centripetal force should have a centripetal acceleration.

$$F_{net} = ma \qquad F_c = \frac{m4\pi^2 r}{T^2}$$

$$a = \frac{F_{net}}{m} \qquad \frac{F_c}{m} = \frac{4\pi^2 r}{T^2}$$

Dividing the net force by the mass gives the acceleration, according to Newton's second law.

$$a_c = \frac{4\pi^2 r}{T^2}$$

7. a. $m = 52.5\ \text{kg}$
$r = 4.1\ \text{m}$
$T = 3.1\ \text{s}$
$a_c = ?$

$$\begin{aligned} v &= \frac{2\pi r}{T} \\ &= \frac{2(\pi)(4.1\ \text{m})}{3.1\ \text{s}} \\ &= 8.31\ \text{m/s} \end{aligned}$$

$$\begin{aligned} a_c &= \frac{v^2}{r} \\ &= \frac{(8.31\ \text{m/s})^2}{4.1\ \text{m}} \\ &= 16.8\ \text{m/s}^2 \\ &= 17\ \text{m/s}^2 \end{aligned}$$

b. $m = 52.5\ \text{kg}$
$r = 4.1\ \text{m}$
$T = 3.1\ \text{s}$
$a_c = ?$

$$\begin{aligned} a_c &= \frac{4\pi^2 r}{T^2} \\ &= \frac{4\pi^2 (4.1\ \text{m})}{(3.1\ \text{s})^2} \\ &= 16.8\ \text{m/s}^2 \\ &= 17\ \text{m/s}^2 \end{aligned}$$

c. $F_c = 884\ \text{N}$
$m = 52.5\ \text{kg}$
$a_c = ?$

$$\begin{aligned} F &= ma \\ F_c &= ma_c \\ a_c &= \frac{F_c}{m} \\ &= \frac{884\ \text{N}}{52.5\ \text{kg}} \\ &= 16.8\ \text{m/s}^2 \\ &= 17\ \text{m/s}^2 \end{aligned}$$

d. The centripetal acceleration in this case is almost double the acceleration due to gravity.

8. The equations in the textbook use a vector notation for v^2. You should not do this because it implies that $\vec{F}_c$ and $\vec{v}$ are in the same direction. Be sure to use the methods outlined in the module booklet.

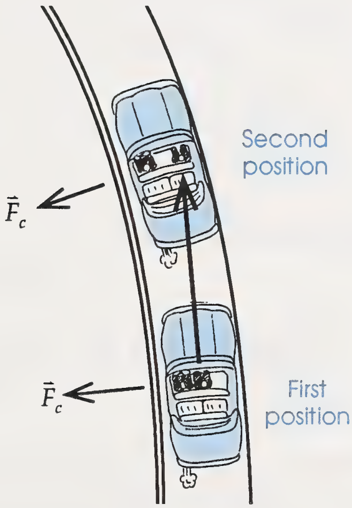
9. Answers to these problems are found on page 668 in your textbook.

10. You would be in the rotating reference frame.

11. As a passenger in the rotating frame, you would likely refer to the centrifugal force when explaining your observations.

12. a.
b.
c.

Answers are shown on the following diagram.



The helicopter is observing the situation from the lab frame of reference.

13. According to Newton's first law, objects tend to maintain their velocity in the absence of external forces. Since the vinyl seat was smooth, there was very little force of friction, and you maintained your velocity in a straight path.

14. No, from the lab frame of reference, you are simply going in a straight line, according to Newton's first law. The car is sliding underneath you and the door eventually gets in the way of your straight path.

15. The friction between the tires and the road keeps the car moving in a curve. In turn, the inside of the car exerts a force on you.

16. Wearing a seatbelt is an important way to keep yourself attached to the vehicle.

17. Textbook question 14. a.:

    1 r/s (one revolution per second) indicates that the period is 1 s.

$$
\begin{aligned}
a_c &= \frac{4\pi^2 r}{T^2} \\
&= \frac{4(3.14)^2(1.3\ \text{m})}{1\ \text{s}^2} \\
&= 51\ \text{m/s}^2
\end{aligned}
$$

Textbook question 14. b.:

The tension on the chain is equal to F_c.

$$
\begin{aligned}
\text{tension} = F_c &= ma_c \\
&= 7.00\ \text{kg}\left(51\ \text{m/s}^2\right) \\
&= 3.6 \times 10^2\ \text{N}
\end{aligned}
$$

Textbook question 16. a.:

The direction of the acceleration on the coin is towards the centre of the record.

Textbook question 16. b.:

The calculations show the acceleration of the coin when placed 5 cm, 10 cm, and 15 cm from the centre of the record.

$$\frac{33\frac{1}{3}\text{ r}}{60\text{ s}} = \frac{1\text{ r}}{x} \qquad x = T = 1.8\text{ s}$$

$r = 5.0$ cm

$$\begin{aligned} a_c &= \frac{4\pi^2 r}{T^2} \\ &= \frac{4(3.14)^2\, 0.05\text{ m}}{(1.8\text{ s})^2} \\ &= 0.61\text{ m/s}^2 \end{aligned}$$

$r = 10$ cm

$$\begin{aligned} a_c &= \frac{4(3.14)^2\, 0.10\text{ m}}{(1.8\text{ s})^2} \\ &= 1.2\text{ m/s}^2 \end{aligned}$$

$r = 15$ cm

$$\begin{aligned} a_c &= \frac{4(3.14)^2\, 0.15\text{ m}}{(1.8\text{ s})^2} \\ &= 1.8\text{ m/s}^2 \end{aligned}$$

Textbook question 16. c.:

The force of friction accelerates the coin.

Textbook question 16. d.:

The coin is most likely to fly off the record at the radius of 15 cm because it requires a greater acceleration to keep it in the circular path.

Textbook question 19. a.:

The calculation shows the speed of the person at the equator.

$r = 6.40 \times 10^6$ m

$T = 24$ h

$= 8.64 \times 10^4$ s

$v = ?$

$$\begin{aligned} v &= \frac{2\pi r}{T} \\ &= \frac{2(3.14)6.40 \times 10^6\text{ m}}{8.64 \times 10^4\text{ s}} \\ &= 465\text{ m/s} \end{aligned}$$

Textbook question 19. b.:

The calculation shows the centripetal force on the person.

$$\begin{aligned} F_c &= \frac{mv^2}{r} \\ &= \frac{97\ \text{kg}(465\ \text{m/s})^2}{6.40 \times 10^6\ \text{m}} \\ &= 3.3\ \text{N} \end{aligned}$$

Textbook question 19. c.:

The calculation shows the weight of the person.

$$\begin{aligned} F_g &= mg \\ &= 97\ \text{kg}\left(9.80\ \text{m/s}^2\right) \\ &= 9.5 \times 10^2\ \text{N} \end{aligned}$$

The weight of the person supplies much more force than is required to keep them attached.

Textbook question 20:

The frictional force supplies the centripetal force.

$$F_f = F_c = \mu F_N \qquad F_N = F_g = mg$$

$$\mu = \frac{F_f}{F_N} = \frac{F_c}{F_N} = \frac{\frac{mv^2}{r}}{mg} = \frac{v^2}{rg}$$

$$\mu = \frac{v^2}{rg}$$

$$v = \sqrt{\mu rg}$$

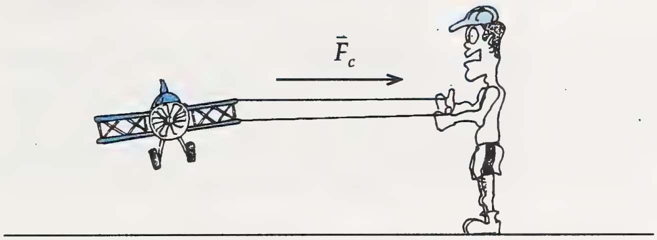
$$v = \sqrt{(0.30)(80\ \text{m})\left(9.80\ \text{m/s}^2\right)}$$

$$v = 15\ \text{m/s}$$

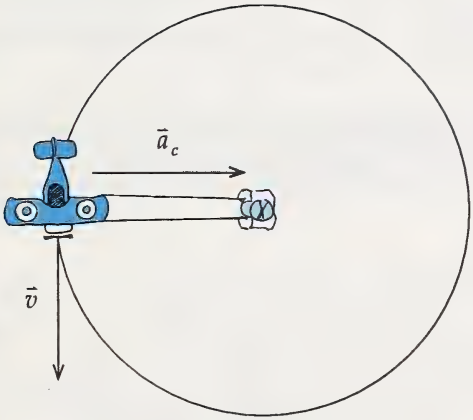
Section 2: Follow-up Activities

Extra Help

1.



2.



3. From the hornet's frame of reference (the rotating frame of reference), there is a centrifugal force pinning it to the wall. From the lab frame of reference, the hornet is simply maintaining its velocity, according to Newton's first law, but the turning plane keeps getting in the way.

4. The speed or magnitude of the velocity is constant, but the direction of the velocity is continually changing. Changing velocity means that the plane must be accelerating. The acceleration acts to the centre of the circular path to turn the velocity vectors. This requires a force that would also be directed to the centre of the path, according to Newton's second law.

5. $m = 2.04\text{ kg}$
   $v = 19.1\text{ m/s}$
   $r = 9.1\text{ m}$
   $T = ?$

Time calculation:

$$v = \frac{2\pi r}{T}$$

$$T = \frac{2\pi r}{v} = \frac{2(\pi)(9.1\text{ m})}{19.1\text{ m/s}} = 2.99\text{ s} = 3.0\text{ s}$$

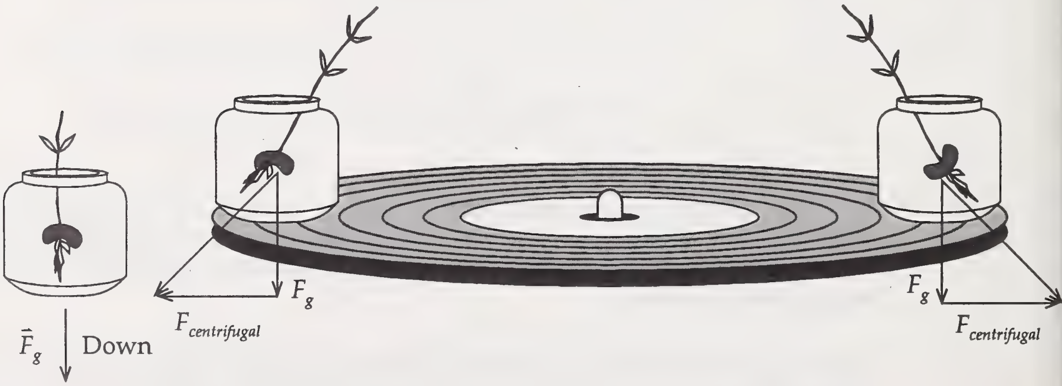
Force calculation:

$$F_c = \frac{mv^2}{r} = \frac{(2.04\text{ kg})(19.1\text{ m/s})^2}{9.1\text{ m}} = 81.8\text{ N} = 82\text{ N}$$

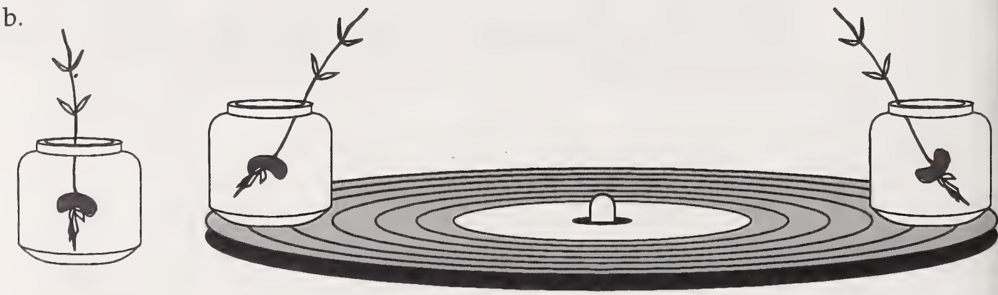
Enrichment

1. a. From the frame of reference of the stationary bean, gravity provides the root with a positive geotropism and it grows down. In other words, the root grows in the direction of the force acting on it. In a similar way, gravity provides the stems with a negative geotropism, and so the stems grow up.

   In the frame of reference of the rotating bean, gravity and the centrifugal force combine to give the illusion that down is now at a new angle. The roots in the rotating frame will grow in the direction of the net force, while the stems will grow in the opposite direction.



   b.



Notice that the stems grow in towards the centre of the turntable (in the opposite direction of the roots) since they grow by a negative geotropism. The roots are shorter and it is more difficult to discern which direction they are actually growing in.

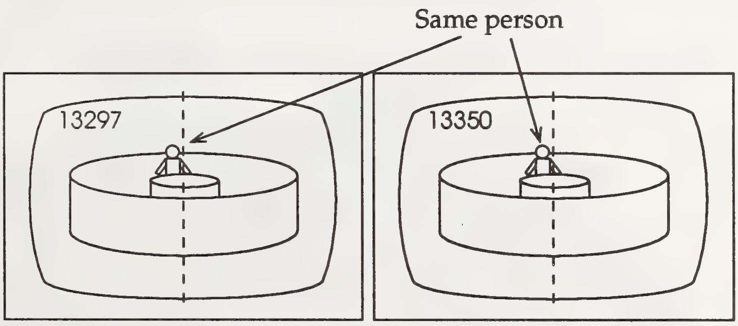
c. Refer to the answer for 1. a.

2. When you are rotating at a constant speed and holding the accelerometer out in front of you, the cork on the string will shift towards you. Because the direction in which the cork points indicates the direction of acceleration, the direction of acceleration is towards you (the centre of the circle). The direction of acceleration is also the same as the direction of the force on the cork, which is also towards you.

3. a. To calculate centripetal acceleration, you need to know the radius and the period of rotation.

   b. $r = 2.1$ m

   c. To calculate the period, use the step function to determine the exact number of frames it takes to have a person make one revolution. A sample calculation is shown for the person in the white shirt and the beige pants.



$$T = 53 \text{ frames}$$
$$= 53 \text{ frames}\left(\frac{1 \text{ second}}{24 \text{ frames}}\right)$$
$$= 2.208 \text{ s}$$

d. $$a_c = \frac{4\pi^2 r}{T^2}$$
$$= \frac{4\pi^2 (2.1\ \text{m})}{(2.208\ \text{s})^2}$$
$$= 17\ \text{m/s}^2$$

e. This sequence is filmed from the rotating frame of reference.

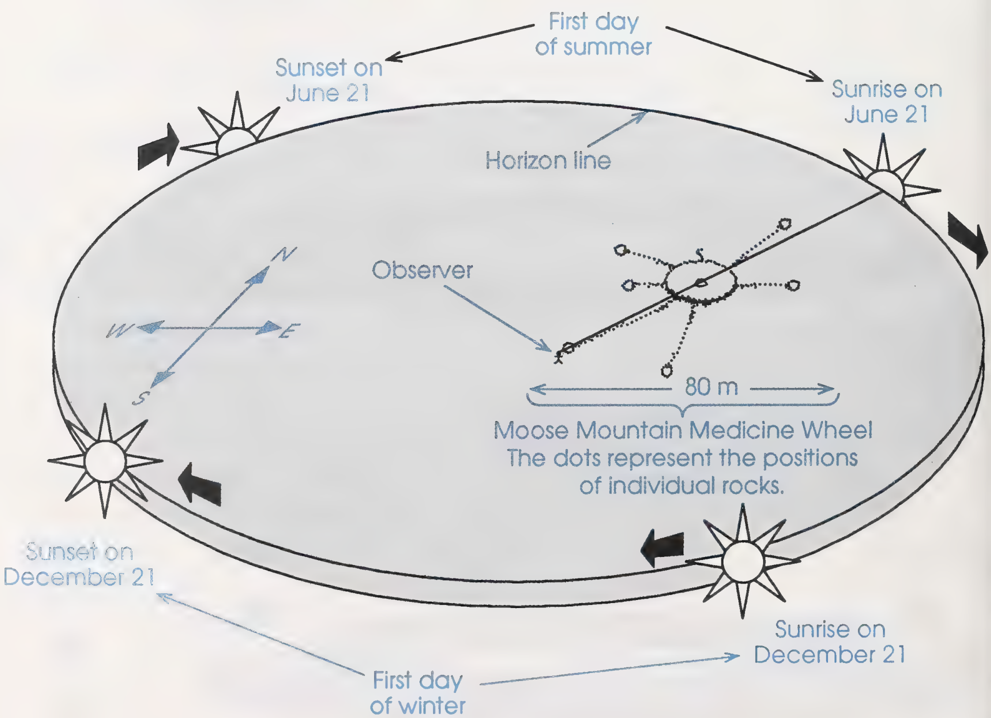
f. The person holding the rope is also in the rotating frame of reference. In this frame, a centrifugal force seems to be responsible for the motion of the ball on the rope.

Section 3: Activity 1

1. The stars can be used to tell direction. The North Star, for example, could be used as a reference point.

2. Life on Earth is dependent on energy from the sun to initiate the flow of energy through the food chain. Without photosythesis there could be no life on Earth.

3.



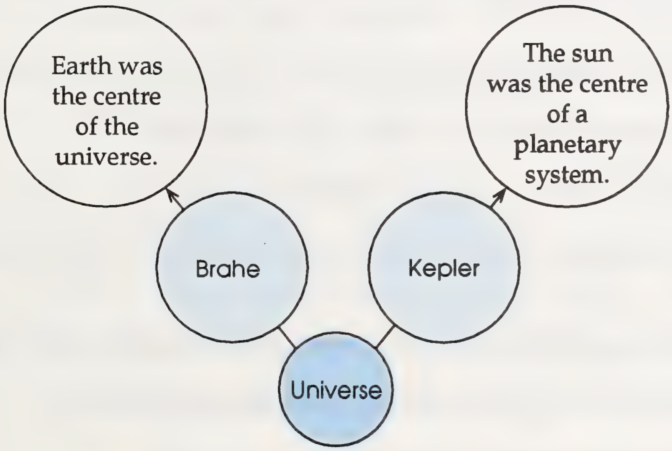
4. The Moose Mountain Medicine Wheel was made so large because the technique of lining up two objects to identify a location is most accurate when the objects are far apart. In the case of the Moose Mountain Medicine Wheel, the position of the rising sun on June 21 is accurate to about two-tenths of one degree.

5. Astronomy was important for ancient peoples because the passing of the seasons required accurate calendars, which in turn required astronomical observations. The passing of the seasons was important for planting and harvesting crops and following the migration patterns of animals.

Section 3: Activity 2

1. Brahe first decided to become an astronomer when he saw an eclipse of the sun. Later on, he also witnessed two planets in conjunction, but was upset that the predicted date of the conjunction was off by two days. After this event, Brahe dedicated his life to making accurate astronomical predictions.

2.



3. Kepler's theories are no longer considered correct.

4. To make a circle instead of an ellipse, use only one thumb tack.

5. A circle is an ellipse with each focus in the same place, in the centre of the circle.

6. The word *radius* implies a circle, but the orbit of Pluto is more like an oval than a circle. Given that the sun is at F_1, sometimes Pluto is at a greater distance than r from the sun, and sometimes it is closer.

7. The orbit of Venus is close to being a perfect circle because each focus is nearly in the same place. Given that the sun is considered to be at F_1, this is very nearly at the centre of the orbit, making *r* close to being the radius of a circle.

8. a. This figure could be considered misleading because the elliptical shape of the orbit is so exaggerated. Not even Pluto has an orbit that is such a flattened oval.

   b. This figure was probably drawn this way to show how an ellipse is different from a circle and to illustrate how planets sweep out equal areas in equal times.

9. Area 2 is equal to Area 1, according to Kepler's second law of planetary motion. Therefore Area 2 is equal to $1.72 \times 10^8 \text{ km}^2$.

10. Planet velocity would be the greatest between A and B because the planet travels faster nearer the focus (the sun).

11. a. The term *period*, as it applies to a planet moving around the sun, is the time taken to complete one revolution.

    b. The period for Earth moving around the sun is one year or 365.25 days.

12. The focus for the two moons is the centre of the planet Jupiter.

13. The unit used to measure the period of Jupiter's moon is days.

14. The units for measuring the radius are not important because they are mathematically eliminated.

15. The focus for the moon's orbit in this example is the centre of the planet Jupiter.

16. The units for the moon's period are mathematically eliminated.

17. These questions are answered on page 668 of your textbook.

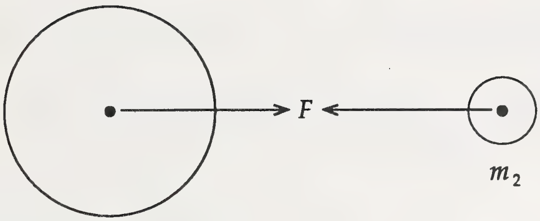
Section 3: Activity 3

1. You need to know the mass of each of the objects and their distance of separation.

2. $F \propto \frac{1}{d^2}$

3. $F \propto m_1 m_2$

4.



5. Since mass varies directly with the force of gravity, tripling the mass will triple the force.

$$F_g = 10\ F \times 3$$
$$F_g = 30\ F$$

The force is now three times its original value.

6. a. Since the force varies inversely with the square of the distance, the force will be changed by the inverse of 4 squared.

$$\begin{aligned} F_g &= 20\ F \times \frac{1}{(4)^2} \\ &= 20\ F \times \frac{1}{16} \\ &= 1.25\ F \end{aligned}$$

The force is now one-sixteenth of its original value.

b. Since the force varies inversely with the square of the distance, the force will be changed by the inverse of $\frac{1}{2}$ squared.

$$\begin{aligned} F_g &= 20\ F \times \frac{1}{\left(\frac{1}{2}\right)^2} \\ &= 20\ F \times 4 \\ &= 80\ F \end{aligned}$$

The force is now four times its original value.

7. $F_g \propto \dfrac{1}{d^2}$ $\qquad$ $F_g \propto m_1 m_2$

$$F_g \propto \frac{m_1 m_2}{d^2}$$

8. Cavendish used lead spheres because they are relatively massive and result in a greater gravitational force of attraction.

9. Cavendish kept the lead spheres enclosed so that the rotational motion of the spheres on the wire would not be influenced by air currents.

10. The small value for G means that the force of gravity calculated for two objects will be smaller. In general, only very massive objects, such as stars or planets or large moons, create a significant gravitational force.

11. $m_1 = 65 \text{ kg}$

$m_2 = 65 \text{ kg}$

$d = 1.0 \text{ m}$

$F_g = ?$

$$F_g = \frac{Gm_1m_2}{r^2} = \frac{\left(6.67\times10^{-11} \text{ N}\bullet\text{m}^2/\text{kg}^2\right)(65 \text{ kg})(65 \text{ kg})}{(1.0 \text{ m})^2}$$
$$= 2.82\times10^{-7} \text{ N}$$
$$= 2.8\times10^{-7} \text{ N}$$

This force is very small. Neither person would notice it.

12. $r_e = 6.37\times10^6 \text{ m}$

$g = 9.80 \text{ m/s}^2$

$G = 6.67\times10^{-11} \text{ N}\bullet\text{m}^2/\text{kg}^2$

$m_e = ?$

$$g = \frac{Gm_e}{r_e^{\,2}}$$
$$m_e = \frac{gr^2}{G}$$
$$= \frac{\left(9.80 \text{ m/s}^2\right)\left(6.37\times10^6 \text{ m}\right)^2}{6.67\times10^{-11} \text{ N}\bullet\text{m}^2/\text{kg}^2}$$
$$= 5.96\times10^{24} \text{ kg}$$

13. As was shown in the previous calculation, the results of the Cavendish experiment allowed the mass of Earth to be calculated.

14. The acceleration due to gravity on Earth is given by $g = \frac{Gm_e}{r_e^{\,2}}$. The mass of the object has no influence on the acceleration due to gravity. The only factors that matter are the mass of the earth, the radius of the earth, and the gravitational constant.

15.

Assuming that the orbits can be treated as circles, a centripetal force is required.

$$\boxed{F_c = \frac{m_p 4\pi^2 r_{ps}}{T^2}}$$

The planet is held in its orbit by the force of gravity, as given by Newton's law of universal gravitation.

$$\boxed{F_g = \frac{Gm_s m_p}{\left(r_{ps}\right)^2}}$$

The force of gravity provides the centripetal force.

$$\boxed{F_c = F_g}$$

Substitute the variables.

$$\boxed{\frac{m_p 4\pi^2 r_{ps}}{T^2} = \frac{Gm_s m_p}{\left(r_{ps}\right)^2}}$$

Cancel the planet's mass.

$$\boxed{\frac{4\pi^2 r_{ps}}{T^2} = \frac{Gm_s}{\left(r_{ps}\right)^2}}$$

Rearrange and solve for the period.

$$\boxed{T = \sqrt{\left(\frac{4\pi^2}{Gm_s}\right) r_{ps}^{\,3}}}$$

16. $r_{ps} = r_{Earth-sun} = 1.4957 \times 10^{11}$ m

$m_s = 1.991 \times 10^{30}$ kg

$T = ?$

$$\frac{(r_{ps})^3}{T^2} = \frac{Gm_s}{4\pi^2}$$

$$T^2 = \frac{(r_{ps})^3 4\pi^2}{Gm_s}$$

$$T = \sqrt{\frac{(1.4957 \times 10^{11})^3 4\pi^2}{(6.67 \times 10^{-11} \text{ N} \bullet \text{m}^2/\text{kg}^2)(1.991 \times 10^{30} \text{ kg})}}$$

$$= 3.154 \times 10^7 \text{ s}$$

$$T = 3.154 \times 10^7 \text{ s}\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)\left(\frac{1 \text{ d}}{24 \text{ h}}\right) = 365 \text{ d}$$

17. $T = 3.154 \times 10^7$ s

$r = 1.4957 \times 10^{11}$ m

$v = ?$

$$v = \frac{2\pi r}{T}$$

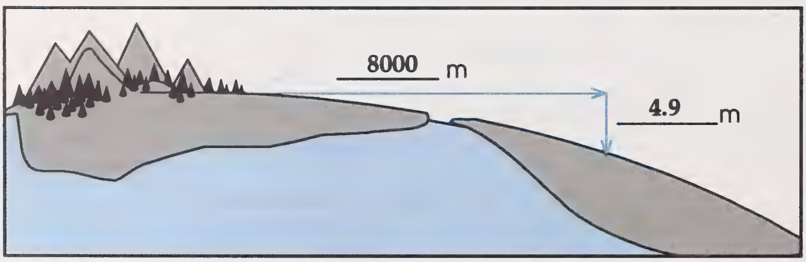
$$= \frac{2\pi(1.4957 \times 10^{11} \text{ m})}{3.154 \times 10^7 \text{ s}}$$

$$= 2.9796 \times 10^4 \text{ m/s}$$

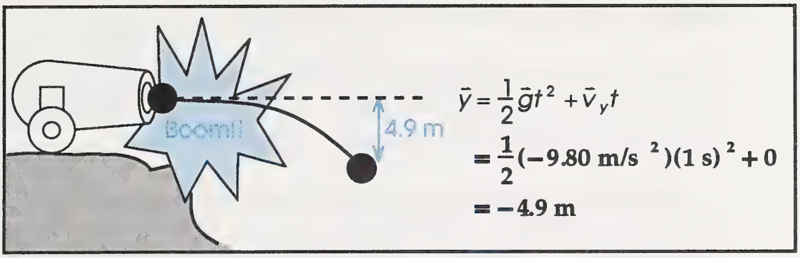
$$= 2.980 \times 10^4 \text{ m/s}$$

Section 3: Activity 4

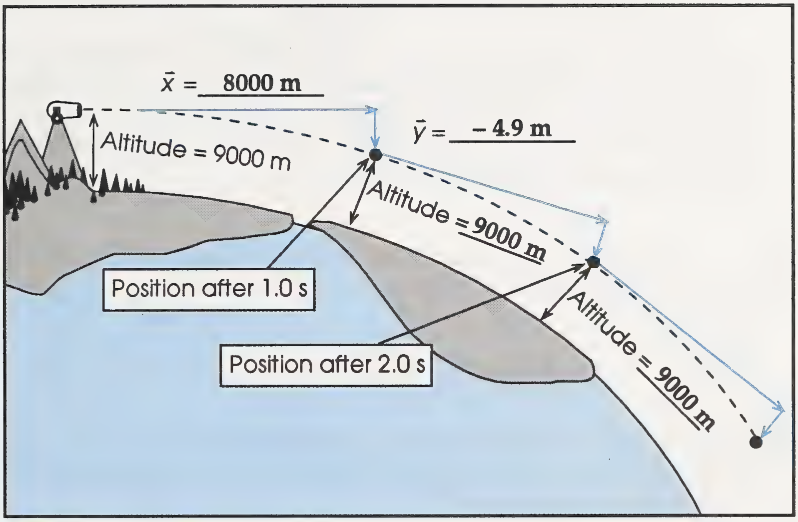
1. a. This diagram has been labelled to show the curvature of the earth.



b.



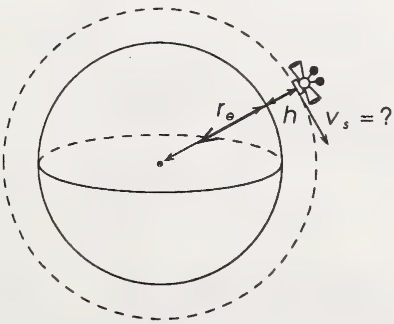
c.



2. The answer to this question is on page 165 of the textbook.

3. The mass of the satellite cancels from the equation and has no effect on the satellite's speed.

4.



The force of gravity is directed to the centre of the circle, like all centripetal forces.

5. Answers to these problems can be found on pages 668 and 669 of the textbook.

6. Weightlessness is the condition in which the object being weighed is in freefall, so the scale being used reports a zero force. The force of gravity is still acting, but it seems to be zero due to the fact that the scale is in free fall, too.

7. In a weightless environment, objects behave as if they have no weight. This means that moving heavy objects around would be much easier than on Earth because the only resistance to motion would be the inertia of the object. The photograph on the cover of the module booklet shows three astronauts manipulating the Intelsat VI satellite.

8. Muscle tone refers to a continued slight contraction of muscles. For example, it is muscle tone that allows you to maintain your posture while sitting in a chair. Even though gravity would tend to pull you to the lowest position possible (on the floor), muscle tone maintains your sitting body position. In a weightless environment, the need for muscle tone would diminish.

9. $$F_g = \frac{Gm_e m_m}{r^2} = \frac{\left(6.67\times10^{-11}\ \text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)\left(7.34\times10^{22}\ \text{kg}\right)}{\left(3.80\times10^{8}\ \text{m}\right)^2}$$
$$= 2.03\times10^{20}\ \text{N}$$

10. Close to Earth, the field is strong, as represented by the long vectors. Far away from Earth, the field is weaker, which means that Earth cannot exert as large a force on a mass that is further away.

11. a. $$g = \frac{Gm_e}{r_e^2} = \frac{\left(6.67\times10^{-11}\ \text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)}{\left(6.37\times10^{6}\ \text{m}\right)^2}$$
$$= 9.83\ \text{N/kg}$$

b. $$g = \frac{Gm_e}{r_s^2} = \frac{\left(6.67\times10^{-11}\ \text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)}{\left(4.23\times10^{7}\ \text{m}\right)^2}$$
$$= 0.223\ \text{N/kg}$$

c. $$g = \frac{Gm_e}{r_m^2} = \frac{\left(6.67\times10^{-11}\ \text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)}{\left(3.80\times10^{8}\ \text{m}\right)^2}$$
$$= 0.002\,76\ \text{N/kg}$$

12. a. $m = 3.2 \times 10^{23}$ kg

$r = 2.43 \times 10^{6}$ m

$g = ?$

$$g = \frac{Gm}{r^2}$$

$$= \frac{\left(6.67 \times 10^{-11}\ \text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(3.2 \times 10^{23}\ \text{kg}\right)}{\left(2.43 \times 10^{6}\ \text{m}\right)^2}$$

$$= 3.6\ \text{N/kg}$$

b. $m = 6.42 \times 10^{23}$ kg

$r = 3.38 \times 10^{6}$ m

$g = ?$

$$g = \frac{Gm}{r^2}$$

$$= \frac{\left(6.67 \times 10^{-11}\ \text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(6.42 \times 10^{23}\ \text{kg}\right)}{\left(3.38 \times 10^{6}\ \text{m}\right)^2}$$

$$= 3.75\ \text{N/kg}$$

c. $m = 1.901 \times 10^{27}$ kg

$r = 69.8 \times 10^{6}$ m

$g = ?$

$$g = \frac{Gm}{r^2}$$

$$= \frac{\left(6.67 \times 10^{-11}\ \text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(1.901 \times 10^{27}\ \text{kg}\right)}{\left(69.8 \times 10^{6}\ \text{m}\right)^2}$$

$$= 26.0\ \text{N/kg}$$

13. $F_g = \left(\frac{Gm}{r^2}\right) m_{stone}$

$F_g = g m_{stone}$

$F_g = m_s g$

a. Earth: $F_g = (4.5\text{ kg})(9.82\text{ N/kg}) = 44\text{ N}$

b. Mercury: $F_g = (4.5\text{ kg})(3.6\text{ N/kg}) = 16\text{ N}$

c. Mars: $F_g = (4.5\text{ kg})(3.75\text{ N/kg}) = 17\text{ N}$

d. Jupiter: $F_g = (4.5\text{ kg})(26.0\text{ N/kg}) = 117\text{ N}$

$= 1.2 \times 10^2\text{ N}$

14. Weight can vary from place to place because weight depends on the strength of the gravitational field. The strength of the gravitational field depends on the mass of the planet and the distance from its centre.

15. $F_g = 3.6\text{ N}$

$m = 370\text{ g or } 0.370\text{ kg}$

$g = ?$

$$F_g = mg$$

$$g = \frac{F_g}{m} = \frac{3.6\text{ N}}{0.370\text{ kg}} = 9.7\text{ N/kg}$$

16. Einstein's concept of gravity involves the idea that a mass can curve the space around itself. When other bodies enter this curved space, they are pulled towards the source.

17. Satellites orbit Earth because Earth is able to curve the space around itself. The effect is similar to that illustrated by the photos of the rubber sheet on the top of page 169 in your textbook. Once caught in the curved space, the satellites becomes trapped in the "well". If a satellite started to slow down, it would spiral towards the planet.

18. The following equations were introduced in this module:

- Projectile Motion:
  (These two are not new equations.)

$$\vec{x} = \vec{v}_x t$$

$$\vec{y} = \vec{v}_y t + \frac{1}{2}\vec{g}t^2$$

- Circular Motion:

$$F_c = \frac{mv^2}{r}$$

$$F_c = \frac{m4\pi^2 r}{T^2}$$

- Gravitation:

$$F_g = \frac{Gm_1 m_2}{r^2}$$

$$g = \frac{Gm_1}{r^2}$$

Section 3: Follow-up Activities

Extra Help

1. The scientist who said all objects accelerate downwards at the same rate was Galileo.

2. When planets are closer to the sun, they travel at higher speeds. When planets are further from the sun, they travel at slower speeds.

3. The point where the sun is positioned with respect to the elliptical orbit is called the focus.

4. Kepler's first law: Planetary orbits are ellipses.
   Kepler's second law: The closer to the sun, the faster the planet moves.
   Kepler's third law: The larger the orbit, the longer the planet takes to go around the sun.

5. During the Apollo moon launch mission, the force of gravity had to be overcome.

6. When the two proportions are combined the following result is obtained.

$$F_g \propto \frac{m_1 m_2}{r^2}$$

7. The value of G is the gravitational constant, which equals $6.67 \times 10^{-11}\ \mathrm{N} \bullet \mathrm{m}^2/\mathrm{kg}^2$.

8. The gravitational field on the moon is weaker than the gravitational field on Earth because the moon has a smaller mass than Earth. Both the hammer and the feather will fall slower than they would on Earth, but they will land on the surface of the moon at the same time because there is no air resistance.

9. The expression $\frac{Gm_e}{r_e^2}$ equals the acceleration due to gravity, or the gravitational field of the earth.

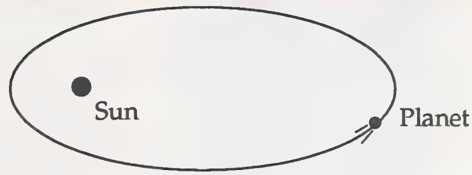
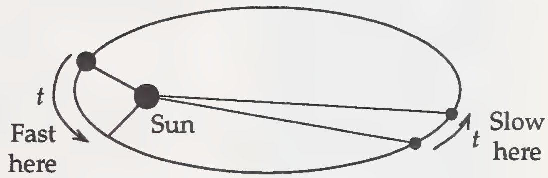
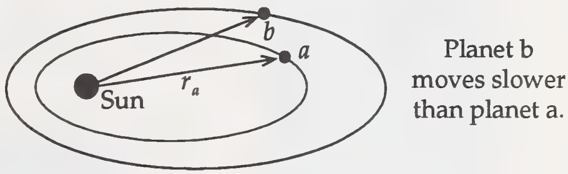
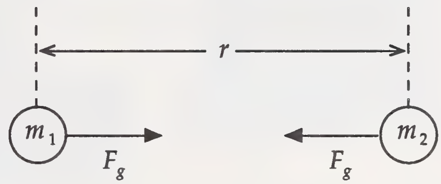
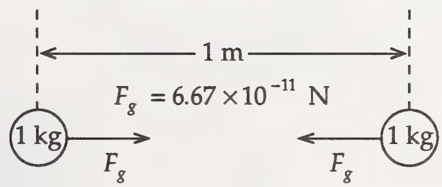
10. When a mass is projected horizontally, the distance that it travels depends on the horizontal velocity of the mass.

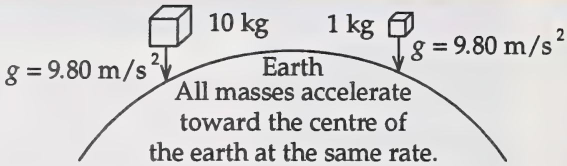
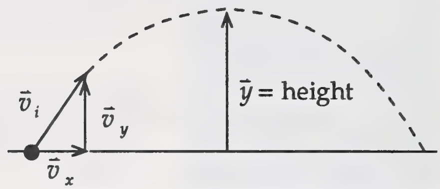
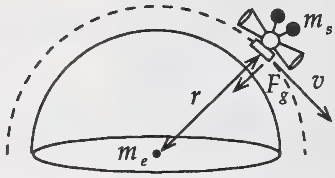
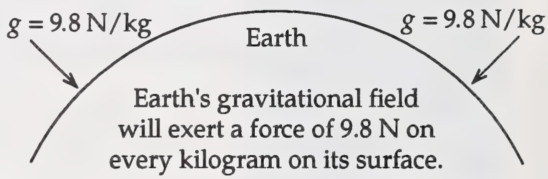
11. The astronaut has difficulty separating the water from the spoon because all masses are weightless and are falling towards the surface of the earth at the same rate.

12. The distance to the moon is equal to 60 Earth radii.

13. The moon will fall one-twentieth of an inch towards the earth in 1 s. In the SI metric system of units, this distance is 1.4 mm.

14.

Summary Chart for Kepler, Newton and Gravitation		
Name of Equation or Idea	**Statement of Equation or Idea**	**Diagram to Illustrate the Meaning of the Variables**
Kepler's First Law	The paths of the planets are ellipses. The sun is centred at the focus of each path.	
Kepler's Second Law	Each planet sweeps out equal areas in equal times.	
Kepler's Third Law	$\left(\frac{t_a}{t_b}\right)^2 = \left(\frac{r_a}{r_b}\right)^2$	
Newton's Law of Universal Gravitation	$F_g = \frac{Gm_1m_2}{r^2}$	
Gravitational Constant	$G = 6.67 \times 10^{-11} \frac{\text{N}\bullet\text{m}^2}{\text{kg}^2}$	

Summary Chart for Kepler, Newton and Gravitation		
Name of Equation or Idea	**Statement of Equation or Idea**	**Diagram to Illustrate the Meaning of the Variables**
Acceleration Due to Gravity	$g = 9.80\ \text{m/s}^2$ $g = \dfrac{Gm_e}{r_e^{\,2}}$ $g = \dfrac{F_g}{m}$	
Vertical Displacement of a Projectile	$\vec{y} = \vec{v}_y t + \frac{1}{2}\vec{g}t^2$	
Centripetal Force for Satellites	$F_c = F_g$ $\dfrac{m_s v^2}{r} = \dfrac{Gm_e m_s}{r^2}$	
Gravitational Field	$g = 9.80\ \text{N/kg}$ $g = \dfrac{Gm_e}{r_e^{\,2}}$ $g = \dfrac{F_g}{m}$	

Enrichment

1. a. The Mayan cities flourished around 1000 A.D. For example, the base of Caracol tower was thought to have been built around 800 A.D.

   b. It is thought that this tower was designed to observe the rising and setting of the sun on the summer solstice (June 21), as well as the most northern setting location of the planet Venus.

   c. The 365-day calendar is based on the number of days that pass from one summer solstice to the next.

   The 584-day calendar is based on observations of Venus. Although Venus takes 225 days to orbit the sun, from the point of view of an observer on Earth, Venus appears to take 584 days to go from its extreme position, east of the sun, back to that position again.

   It is interesting to note that five Venus years exactly equal eight earth years, which is 2920 days. Every 2920 days, Venus will return to the same place in the sky at the same time of year.

   d. The Mayan civilization relied on agriculture for their source of food. It was important to have accurate calendars so that they could carefully track the seasons and know the best time to plant maize and other crops.

2. a.

$$\boxed{F_c = F_g}$$

m_s = mass of the sun
m_p = mass of the planet

$$\boxed{\frac{m_p 4\pi^2 r}{T^2} = \frac{Gm_s m_p}{r^2}}$$

Cancel the mass of the planet (m_p) and rearrange.

$$\boxed{\frac{r^3}{T^2} = \frac{Gm_s}{4\pi^2}}$$

b.

$$\frac{Gm_s}{4\pi^2} = \frac{\left(6.67\times10^{-11}\ \frac{\text{N}\bullet\text{m}^2}{\text{kg}^2}\right)\left(1.991\times10^{30}\ \text{kg}\right)}{4\pi^2}$$

$$= 3.36\times10^{18}\ \frac{\text{N}\bullet\text{m}^2}{\text{kg}}$$

$$= 3.36\times10^{18}\ \frac{\left(\cancel{\text{kg}}\frac{\text{m}}{\text{s}^2}\right)\bullet\text{m}^2}{\cancel{\text{kg}}}$$

$$= 3.36\times10^{18}\ \frac{\text{m}^3}{\text{s}^2}$$

c. Earth: $T = 365\ \text{d} = 3.15\times10^7\ \text{s}$

$$\frac{r^3}{T^2} = \frac{\left(1.4957\times10^{11}\ \text{m}\right)^3}{\left(3.15\times10^7\ \text{s}\right)^2}$$

$$= 3.37\times10^{18}\ \frac{\text{m}^3}{\text{s}^2}$$

Venus: $T = 225\ \text{d} = 1.94\times10^7\ \text{s}$

$$\frac{r^3}{T^2} = \frac{\left(1.081\times10^{11}\ \text{m}\right)^3}{\left(1.94\times10^7\ \text{s}\right)^2}$$

$$= 3.36\times10^{18}\ \frac{\text{m}^3}{\text{s}^2}$$

Mars: $T = 687\ \text{d} = 5.94\times10^7\ \text{s}$

$$\frac{r^3}{T^2} = \frac{\left(2.278\times10^{11}\ \text{m}\right)^3}{\left(5.94\times10^7\ \text{s}\right)^2}$$

$$= 3.35\times10^{18}\ \frac{\text{m}^3}{\text{s}^2}$$

d. Yes, the value of $\frac{r^3}{T^2}$ is very close or equal to the value of $\frac{Gm_s}{4\pi^2}$ for each planet.

Section 1: Activity 2 Investigation: A Projectile Marble

Physics 20

Student Module Booklet
Module 3

L.R.D.C.
Producer

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